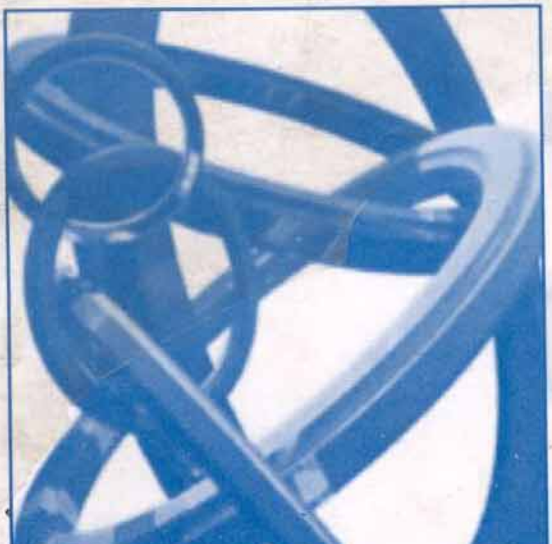
Key to BUSINESS MATHEMATICS

I.COM. PART -1



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KEY

TO

BUSINESS

MATHEMATICS

For

I. Com

(Part I) Students

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EXERCISE NO. 1

SET – A

-:1.1:-

Simplify the following ratios

i)	a)	40:32	ii)	a)	4:12:16
	b)	48:80		b)	25:75:35
	c)	24:16		c)	13:39:52
	d)	12:32		d)	18:54:90

SOLUTION

	(i)		(ii)
a)	40 : 32 10 : 8 5 : 4	a)	4 : 12 : 16 1 : 3 : 4
b)	48 : 80 12 : 20 3 : 5	b)	25 : 75 : 35 5 : 15 : 7
c)	24 : 16 3 : 2	c)	13 : 29 : 52 1 : 3 : 4
d)	12 : 32 3 : 8	d)	18 : 54 : 90 2 : 6 : 10 1 : 3 : 5

-:1.2:-

Find the missing quantities in each of the following.

a) $\frac{4800}{600} = \frac{2000}{x}$ b) $\frac{648}{108} = \frac{x}{50}$

c) $\frac{45.6}{x} = \frac{76.5}{25.5}$ d) $\frac{x}{8\frac{3}{4}} = \frac{6}{14}$

SOLUTION

a)

$$\frac{4800}{600} = \frac{2000}{x}$$

$$4800:600::2000:x$$

$$x \times 4800 = 600 \times 2000$$

$$x = \frac{600 \times 2000}{4800} = 250$$

b)
$$\frac{648}{108}=\frac{x}{50}$$
$$648:108::x:50$$
$$648\times 50=108\times x$$
$$108x=648\times 50$$
$$x=\frac{648\times 50}{108}=300$$

c)
$$\frac{45.6}{x}=\frac{76.5}{25.5}$$
$$45.6:x::76.5:25.5$$
$$45.6\times 25.5=76.5\times x$$
$$76.5x=45.6\times 25.5$$
$$x=\frac{45.6\times 25.5}{76.5}=15.2$$

d)
$$\frac{x}{8\frac{3}{4}}=\frac{6}{4}\Rightarrow\frac{x}{\frac{35}{4}}=\frac{6}{14}$$
$$\frac{4x}{35}=\frac{6}{14}$$
$$4x:35=6:4$$
$$56x=35\times 6$$
$$16x=35\times 6$$
$$x=\frac{35\times 6}{56}=3\frac{3}{4}$$

-:1.3:-

Distribute a stock of 6000 electric fans to the three dealers in ratio 3:5:4.

SOLUTION

Total stock of electric fans = 6000 fan
Ratio of fans to three dealers

3 : 5 : 4

Sum of ratios = 3 + 5 + 4 = 12

Hira

$$\text{Share of first dealer} = \frac{3}{12} \times 6000 = 1500 \text{ fans}$$

$$\text{Share of second dealer} = \frac{5}{12} \times 6000 = 2500 \text{ fans}$$

$$\text{Share of third dealer} = \frac{4}{12} \times 6000 = 2000 \text{ fans}$$

-:1.4:-

Three people invest Rs. 900, Rs. 600, Rs. 300 respectively in a business. How should they share out profit of Rs. 900.

SOLUTION

The ratio of investment of three person

Rs. 900 : Rs. 600 : Rs. 300

9 : 6 : 3

3 : 2 : 1

Sum of ratios $= 3+2+1 = 6$

Total profit = Rs. 900

$$\text{Share of first person} = \frac{3}{6} \times 900 = \text{Rs. } 450$$

$$\text{Share of 2nd person} = \frac{2}{6} \times 900 = \text{Rs. } 300$$

$$\text{Share of thrid person} = \frac{1}{6} \times 900 = \text{Rs. } 150$$

-:1.5:-

If Rs. 750 is received as the profit on an investment of Rs. 2500. What return might be expected on an investment of Rs. 3850.

SOLUTION

Let x be the expected profit on an investment of Rs. 3850. The estimated value is greater, so we form the proportion with smaller value first.

Profit (Rs.)	Investment (Rs.)
750	2500
x	3850

It is a direct proportion. Applying the fundamental principle of proportion.

$$750 \times 3850 = x \times 2500$$
$$x \times 2500 = 750 \times 3850$$
$$x = \frac{750 \times 3850}{2500}$$
$$= \text{Rs. } 1155$$

-:1.6:-

In a partnership, A invested Rs. 8000, B Rs. 6000 and C Rs. 5000. If their profits totaled Rs. 12000, how much did each receive if the profits were divided in the ratio of their investment?

SOLUTION

The ratio of investment of three person A, B and C is

Rs. 8000 : Rs. 6000 : Rs. 5000

8 : 6 : 5

Sum of ratios $= 8 + 6 + 5 = 19$

$$\text{A's Share of profit} = \frac{8}{19} \times 12000 = \text{Rs.} 5052$$

$$\text{B's Share of profit} = \frac{6}{18} \times 12000 = \text{Rs.} 3789.47$$

$$\text{C's Share of profit} = \frac{5}{19} \times 12000 = \text{Rs.} 3157.90$$

-:1.7:-

The cost of building 1.5 miles of a certain highway was Rs. 420,000; what was the cost of 4.5 miles of that highway.

SOLUTION

Let x be the cost of building 4.5 miles highway. The estimated value is greater so we form the proportion with first ratio smaller.

Distance (miles)	Cost (Rs.)
1.5	420000
4.5	x

1.5 : 4.5 : 420000 : x

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$1.5 \times x = 4.5 \times 420000$$

$$x = \frac{4.5 \times 420000}{1.5}$$

$$= \text{Rs.}12,60,000$$

-:1.8:-

A cylindrical tank 24 feet high, now holds 375 gallons of water 9 feet deep. How many gallons will it hold when full.

SOLUTION

Let x be the number of gallons of water to fill the full tank. The estimated value is greater, we form the proportion with first ratio smaller.

Height (feet)	Water (in gallon)
9	375
24	x

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$9 \times x = 24 \times 375$$

$$x = \frac{24 \times 375}{9} = 1000 \text{ gallons}$$

-:1.9:-

A jet airplane traveled 100 miles in 9 minutes. To the nearest mile, what was its rate per hours?

SOLUTION

Let x be the distance travelled in one hour or in 60 minutes. The estimated value is greater, so we form the proportion as:

Time (in minutes)	Distance (miles)
9	100
60	x

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$9 \times x = 60 \times 100$$

$$x = \frac{60 \times 100}{9} = 666.67 \text{ miles}$$

-:1.10:-

A space ship in orbit has an average speed of 11400 m.p.h. At that rate, how many minutes would it take to travel 3000 miles?

SOLUTION

Let x be the minutes taken to travel 3000 miles. The estimated value is smaller, so we form the proportion as:

Distance (miles)	Time (in minutes)
11400	60
3000	x

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$11400 \times x = 3000 \times 60$$

$$x = \frac{3000 \times 60}{11400}$$

$$= 15.79 \text{ miles}$$

-:1.11:-

A train covers 144 km distance in two hours. What distance will at cover in 50 minutes with the same speed.

SOLUTION

Let x be the distance covered in 50 minutes. The estimated value (distance) is smaller, so we form the proportion with first ratio greater

Distance (km)	Time (in min)
144	120
x	50

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$144 \times 50 = x \times 120$$

$$x \times 120 = 144 \times 50$$

$$x = \frac{144 \times 50}{120} = 60 \text{ km}$$

144 : x :: 120 : 50

-:1.12:-

A bus travels 200 kilometers in 3 hours. How much time is needed for a journey of 450 kilometers.

SOLUTION

Let x be the time for a journey of 450 km.
The ratio of distance to time is

$$200 : 3$$

Hence $$200 : 3 :: 450 : x$$

Applying the fundamental law of proportion, we get

$$200x = 3 \times 450$$

$$x = \frac{3 \times 450}{200} = 6\frac{3}{4} \text{ hours}$$

-:1.13:-

Six men can paint a house in four days. How long would it take to paint the house if two men are employed.

SOLUTION

Let x be the number of days when 2 men are employed to paint the house. The number of men are decreased but the number of days increased. This is the inverse proportion. The proportion is:

Men	Days
6	4
2	X

$$6 : 2 : : 4 : x$$

For inverse proportion, we invert the first ratio and write the proportion as:

$$2 : 5 : : 4 : x \text{ (inverse)}$$

Applying the fundamental principle of proportion

$$2 \times x = 6 \times 4$$

$$x = \frac{6 \times 4}{2} = 12 \text{ days}$$

-:1.14:-

A factory can produce 72 washing machines in 9 days.

a) How many machines can it produce in the following days.

i) 1 day ii) 10 days iii) 24 days

b) **How may days will it take to produce the following numbers of machines.**

i) 8 machines ii) 16 machines iii) 126 machines

SOLUTION

(a)

(i) Let x be the number of machines produced in one day.

The estimated value is smaller, so we form the proportion with first ratio as greater

Machines	Days
72	9
x	1

$$\frac{72}{x} = \frac{9}{1}$$

$$72 : x :: 9 : 1$$

Applying the fundamental principle of proportion,

$$72 \times 1 = x \times 9$$

$$9x = 72$$

$$x = 8 \text{ machines}$$

(ii) Let x be the number of machines produced in 10 days.

The estimated value is greater, so we form the proportion with first ratio as smaller.

Machines	Days
9	72
10	x

$$\frac{9}{10} = \frac{72}{x}$$

$$9 : 10 :: 72 : x$$

Applying the fundamental principle of proportion

$$x \times 9 = 720$$

$$x = \frac{720}{9} = 80 \text{ machines}$$

(iii) Let x be the number of machines to be produced in 24 days.

Days	Machines
9	72
24	x

$$\frac{9}{24} = \frac{72}{x}$$

$$9 : 24 :: 72 : x$$

Applying the fundamental principle of proportion

$$9x = 24 \times 72$$

$$x = 192 \text{ machines}$$

(b)

(i) Let x be the number of days to produce 8 machines.
The estimated value is smaller, the first ratio will be greater.

Machines	Days
72	9
8	x

$$\frac{72}{8} = \frac{9}{x}$$

$$72 : 8 :: 9 : x$$

Applying the fundamental principle of proportion

$$72x = 8 \times 9$$

$$x = 1 \text{ day}$$

(ii) Let x be the number of days to produce 16 machines.
The estimated value is smaller, so we form the proportion with first ratio as greater.

Machines	Days
72	9
16	x

$$\frac{72}{16} = \frac{9}{x}$$

$$72 : 16 :: 9 : x$$

Applying the fundamental principle of proportion

$$72x = 16 \times 9$$

$$72x = 144$$

$$x = 2 \text{ days}$$

(iii) Let x be the number of day to produce 126 machines.

The estimated value is greater, so we form the proportion with first ratio as smaller.

Days	Machines
9	72
x	126

$$\frac{9}{x}=\frac{72}{126}$$

$$9 : x :: 72 : 126$$

Applying the fundamental principle of proportion

$$9\times126=x\times72$$

$$72x=9\times126$$

$$x=15\frac{3}{4}\text{ days}$$

-:1.15:-

A woman can walk 48 km in 6 hours.

a) How long will she take to walk the following distances:

i) 16 km ii) 40 km iii) 143 km

b) How far can she walk in

i) 2 hrs ii) 5 hrs iii) 7 ½ hrs

SOLUTION

(i) The estimated value (time) is smaller, so we form the proportion with first ratio as greater. Let x be the time used to walk 16 km.

Distance	Time
48	6
16	x

$$48 : 16 :: 6 : x$$

$$48x=16\times6$$

$$x=\frac{96}{48}=2\text{ hours}$$

(ii) The estimated value (time) is smaller, so we form the proportion with first ratio as greater. Let x be the time used to walk 40 km.

Distance	Time
48	6
40	x

$$\frac{48}{40} = \frac{6}{x}$$

$$48 : 40 :: 6 : x$$

Applying the fundamental principle of proportion

$$48x = 40 \times 6$$

$$x = \frac{40 \times 6}{48} = 5 \text{ hours}$$

(iii) The estimated value (time) is greater, so we form the proportion with first ratio as smaller. Let x be the time used to walk 143 km.

Time	Distance
6	48
x	143

$$\frac{6}{x} = \frac{48}{143}$$

$$6 : x :: 48 : 143$$

Applying the fundamental principle of proportion

$$6 \times 143 = x \times 48$$

$$48x = 6 \times 143$$

$$x = \frac{6 \times 143}{48} = 17\frac{7}{8} \text{ hours}$$

(b)

(i) The estimated value (distance) is smaller, so we form the proportion with first ratio as greater. Let x be the distance to be covered in two hours.

Distance	Time
48	6
x	2

$$\frac{48}{x} = \frac{6}{2}$$

$$48 : x :: 6 : 2$$

Applying the fundamental principle of proportion

$$48 \times 2 = 6x$$

$$6x = 48 \times 2$$

$$x = 16 \text{ km}$$

(ii) The estimated value (distance) is smaller, so we form the proportion with first ratio as greater. Let x be the distance to be covered in 5 hours.

Distance	Time
48	6
x	5

$$\frac{48}{x} = \frac{6}{5}$$

$$48 : x :: 6 : 5$$

Applying the fundamental principle of proportion

$$48 \times 5 = 26x$$

$$6x = 48 \times 5$$

$$x = \frac{45 \times 5}{6} = 40 \text{ km}$$

(iii) The estimated value (distance) is greater, so we form the proportion with first ratio as smaller. Let x be the distance to be covered in 7½ hours.

Time	Distance
6	48
7½	x

$$\frac{6}{7\frac{1}{2}} = \frac{48}{x}$$

$$6 : \frac{15}{2} :: 48 : x$$

Applying the fundamental principle of proportion

$$6x = \frac{15}{2} \times 48$$

$$6x = 15 \times 24$$

$$x = \frac{15 \times 24}{6} = 60 \text{ km}$$

-:1.16:-

A well produces 2846 cu ft water in 3 ½ hr. How many hours will it take the well to produce 14230 cu ft of water.

SOLUTION

The estimated value (time) is greater, so we form the proportion with first ratio as smaller. Let x be the time used to produce required amount of water.

Time (hrs)	Water in cu ft
3½	2846
x	14230

$$\frac{3.5}{x} = \frac{2846}{14230}$$

$$3.5 : x :: 2846 : 14230$$

Applying the fundamental principle of proportion

$$3.5 \times 14230 = x \times 2846$$

$$2846x = 3.5 \times 14230$$

$$x = \frac{3.5 \times 14230}{2846} = 17\frac{1}{2} \text{ hrs}$$

-:1.17:-

A bus runs 100 km in 8 litres of diesel. How much diesel is needed to run 250 km.

SOLUTION

The estimated value diesel in litres is greater, so we form the proportion with first ratio as smaller. Let x be the amount of diesel required.

Diesel (in litres)	Distance (km)
8	100
x	250

$$\frac{8}{x} = \frac{100}{250}$$

$$8 : x :: 100 : 250$$

Applying the fundamental principle of proportion

$$8 \times 250 = x \times 100$$

$$100x = 250 \times 8$$

$$x = \frac{250 \times 8}{100} = 20 \text{ litres}$$

-:1.18:-

If 15 dozens of eggs cost Rs. 202.50. How much will 96 eggs cost?

SOLUTION

Let x be the cost for 96 eggs that is 8 dozen eggs. The estimated value is smaller, so we form the proportion with first ratio greater

$$202.50 : X :: 15 : 8$$

Cost (in Rs.)	Eggs (in dozen
202.50	15
x	8

It is the problem of direct proportion. Applying the fundamental principle of proportion.

$$202.50 \times 8 = x \times 15$$

$$x \times 15 = 202.50 \times 8$$

$$x = \frac{202.50 \times 8}{15} = \text{Rs.}108$$

-:1.19:-

8 men take 48 hours to dig a garden.

a) **How long will be taken by the following numbers of men.**

i) 12 men ii) 32 men iii) 40 men

b) **How many men will be required to dig the garden in the following times.**

i) 24 hrs. ii) 96 hrs. iii) 32 hrs.

SOLUTION

(a)

(i) Let x be the time to be used to dig the garden, then

In this problem, 12 men will dig the garden in less than 48 hours.

Hours	Men
48	8
X	12

Here one quantity is decreased while the other is increased. So this is the problem of inverse proportion.

$$48 : x :: 8 : 12$$

As it is an inverse proportion, we invert the first ratio and write the proportion as:

$$x : 48 :: 8 : 12$$

Applying the fundamental principle of proportion

$$12x = 48 \times 8$$

$$x = \frac{48 \times 8}{12} = 32 \text{ hrs}$$

(ii) Let x be the time to be used to dig the garden.

In this problem, 32 men will dig the garden in less time.

Hours	Men
48	8
x	32

Here one quantity is decreased when the other is increased. It is the problem of inverse proportion.

$$48 : x :: 8 : 32$$

As it is an inverse proportion, we invert the first ratio and write the proportion as:

$$x : 48 :: 8 : 32$$

Applying the fundamental principle of proportion

$$32x = 48 \times 8$$

$$x = \frac{48 \times 8}{32} = 12 \text{ hrs}$$

(iii) Let x be the time to be used to dig the garden.

Hours	Men
48	8
X	40

$$48 : x :: 8 : 40$$

As it is an inverse proportion, we invert the first ratio and write the proportion as:

$$x : 48 :: 8 : 40$$

Applying the fundamental principle of proportion

$$40x = 48 \times 8$$

$$x = \frac{48 \times 8}{40} = 9.6 \text{ hrs}$$

(b)

Let x be the number of men required to dig the garden.

(i)

Men	Hours
8	48
x	24

Decrease in time will increase the number of men. It is the problem of inverse proportion.

$$8 : x :: 48 : 24$$

Invert the first ratio and write the proportion as:

$$x : 8 :: 48 : 24$$

Applying the fundamental principle of proportion

$$24x = 8 \times 48$$

$$x = \frac{8 \times 48}{24} = 16 \text{ men}$$

(ii)

Hours	Men
48	8
96	x

$$48 : 96 :: 8 : x$$

As it the problem of inverse proportion. Invert the first ratio and write the proportion as:

$$96 : 48 :: 8 : x$$

Applying the fundamental principle of proportion

$$96x = 48 \times 8$$

$$x = \frac{48 \times 8}{96} = 4 \text{ men}$$

(iii)

Men	Hours
8	48
x	32

$$8 : x :: 48 : 32$$

It is the problem of inverse proportion, invert the first ratio

$$x : 8 :: 48 : 32$$

Applying the fundamental principle of proportion

$$32x = 8 \times 48$$

$$x = \frac{8 \times 48}{32} = 12 \text{ men}$$

-:1.20:-

A car take 8 hrs. To do a journey at 90 km/h.

a) How long would it take at the following speeds.

i) 144 km/h ii) 80 km/h iii) 60 km/h

b) What speed would be required to do it in the following times.

i) 10 hrs ii) 18 hrs iii) 6 hrs

SOLUTION

(a)

Let x be the time used at the given speed.

(i)

Speed (km/h)	Time (hrs)
90	8
144	x

$$90 : 144 :: 8 : x$$

As it is an inverse proportion

$$144 : 90 :: 8 : x$$

Applying the fundamental principle of proportion

$$144x = 90 \times 8$$

$$x = \frac{90 \times 8}{144} = 5 \text{ hrs}$$

(ii)

Time (hrs)	Speed (km/h)
8	90
x	80

$$8 : x :: 90 : 80$$

As it is an inverse proportion

$$x : 8 :: 90 : 80$$

Applying the fundamental principle of proportion

$$80x = 8 \times 90$$

$$x = \frac{8 \times 90}{80} = 9 \text{ hrs}$$

(iii)

Time (hrs)	Speed (km/h)
8	90
x	60

$$8 : x :: 90 : 60$$

As it is an inverse proportion

$$x : 8 :: 90 : 60$$

Applying the fundamental principle of proportion

$$60x = 8 \times 90$$

$$x = \frac{8 \times 90}{60} = 12 \text{ hrs}$$

(b)

Let x be the speed to do the journey in the required time.

(i)

Speed (km/h)	Time (hrs)
90	8
x	10

$$90 : x :: 8 : 10$$

As it is an inverse proportion

$$x : 90 :: 8 : 10$$

Applying the fundamental principle of proportion

$$10x = 90 \times 8$$

$$x = \frac{90 \times 8}{10} = 72 \text{ km / h}$$

(ii)

Speed (km/h)	Time (hrs)
90	8
x	18

$$90 : x :: 8 : 18$$

As it is an inverse proportion, so

$$x : 90 :: 8 : 18$$

Applying the fundamental principle of proportion

$$18x = 90 \times 8$$

$$x = \frac{90 \times 8}{18} = 40 \text{ km / h}$$

(iii)

Time (hrs)	Speed (km/h)
8	90
6	x

$$8 : 6 :: 90 : x$$

As it is an inverse proportion

$$6 : 8 :: 90 : x$$

Applying the fundamental principle of proportion

$$6x = 8 \times 90$$

$$x = \frac{8 \times 90}{6} = 120 \text{ km / h}$$

-:1.21:-

A Suzuki car runs 150 km in 10 litres of petrol. How much petrol would be used to go 280 km?

SOLUTION

Let x be the petrol in litres to go 280 km.

Petrol (litres)	Distance (km)
10	150
x	280

$$10 : x :: 150 : 280$$

Is the problem of direct proportion.

Applying the fundamental principle of proportion

$$10 \times 280 = x \times 150$$

$$150x = 10 \times 280$$

$$x = \frac{10 \times 280}{150} = 18\frac{2}{3} \text{ litres}$$

-:1.22:-

Supper express takes 4¼ hours at a speed of 60 km/h to go from Faisalabad to Multan. How fast must the train travel for the same distance in 3½ hours.

SOLUTION

Let x be the speed of supper express to reach Multan in required time.

Time (hrs)	Speed (km/h)
$\frac{17}{4}$	60
$\frac{7}{2}$	x

$$\frac{17}{4} : \frac{7}{2} :: 60 : x$$

This is the problem of inverse proportion.

$$\frac{7}{2} : \frac{17}{4} :: 60 : x$$

Applying the fundamental principle of proportion

$$\frac{7}{2}x = \frac{17}{4} \times 60$$

$$\frac{7}{2}x = 17 \times 15$$

$$x = \frac{(17 \times 15)2}{7} = 73 \text{ km / h approxi.}$$

-:1.23:-

In a factory 12 men complete the work in 20 days, how long it will take to complete the same work by 8 workers.

SOLUTION

Let x be the number of days to compile the work with required workers.

Men	Days
12	20
8	x

$$12 : 8 :: 20 : x$$

It is the problem of inverse proportion.

$$8 : 12 :: 20 : x$$

Applying the fundamental principle of proportion

$$8x = 12 \times 20$$

$$x = \frac{12 \times 20}{8} = 30 \text{ men}$$

-:1.24:-

A group of 42 men can construct a house in 60 days working 8 hours a day. How many days are required to construct the same house by 60 workers, if they work 7 hours day.

SOLUTION

Let x be the number of days to complete the house by 60 workers, if they work 7 hours a day.

Ratio of men = 60 : 42 (inverse)

Ratio of hours = 7 : 8 (inverse)

In proportion

$$\left.\begin{matrix} 60 : 42 \\ 7 : 8 \end{matrix}\right\} :: 60 : x$$

By applying the fundamental principle of proportion

$$x \times 7 \times 60 = 60 \times 42 \times 8$$

$$420x = 60 \times 42 \times 8$$

$$x = \frac{60 \times 42 \times 8}{420} = 48 \text{ days}$$

-:1.25:-

100 men finished a job in 12 days working 6 hours a day. How many men will do the same job in 15 days to work 8 hours a day.

SOLUTION

Let x be the number of men who finish the work in 15 days to work 8 hours a day.

Ratio of days = 15 : 12 (inverse)

Ratio of men = 100 : 6 (inverse)

In proportion

$$\left.\begin{matrix} 15 : 12 \\ 8 : 6 \end{matrix}\right\} :: 100 : x$$

By applying the fundamental principle of proportion

$$x \times 8 \times 15 = 100 \times 12 \times 6$$

$$x = \frac{100 \times 12 \times 6}{8 \times 15} = 60 \text{ men}$$

-:1.26:-

In a hotel 10 men stay 16 days costs Rs. 8000. How many days, 15 men can stay with the amount Rs. 9750.

SOLUTION

Let x be the number of days to stay in hotel 15 men with amount Rs. 9750.

Ratio of men = 15 : 10 (inverse)
Ratio of amount = 8000 : 9750 (direct)
Ratio of days = 16 : x

In proportion

$$\left.\begin{matrix}15 : 10\\ 8000 : 9750\end{matrix}\right\} :: 16 : x$$

By applying the fundamental principle of proportion

$$x \times 8000 \times 15 = 16 \times 10 \times 9750$$

$$x = \frac{16 \times 10 \times 9750}{8000 \times 15} = 13 \text{ days}$$

-:1.27:-

50 men working 8 hours a day can complete a building in 35 days. How many hours a day. 70 men work to complete the same work in 25 days.

SOLUTION

Let x be the number of hours a day to complete the building by 60 men to work 25 days.

Ratio of men = 70 : 50 (inverse)

Ratio of days = 25 : 35 (inverse)

Ratio of hours = 8 : x

In proportion

$$\left.\begin{matrix}70 : 50\\ 25 : 35\end{matrix}\right\} :: 8 : x$$

By applying the fundamental principle of proportion

$$x \times 25 \times 70 = 8 \times 50 \times 25$$

$$x = \frac{8 \times 50 \times 35}{25 \times 70} = 8 \text{ hours}$$

-:1.28:-

25 workers complete a job in 20 days working 8 hours a day. In how many days 20 workers will complete the work for 10 hours a day?

SOLUTION

Let x be the number of days to finish the work by 20 workers working 10 hours a day.

Ratio of men = 20 : 25 (inverse)

Ratio of hours = 10 : 8 (inverse)

Ratio of days = 20 : x

In proportion

$$\left.\begin{matrix}20:20\\10:8\end{matrix}\right\} :: 20:x$$

Applying the fundamental principle of proportion

$$x \times 10 \times 20 = 20 \times 25 \times 8$$

$$x = \frac{20 \times 25 \times 8}{10 \times 20} = 20 \text{ days}$$

-:1.29:-

In a factory, a group of 50 workers 10 hours daily to make 500 units in 10 days. There is demand of 720 units in 12 days. How many workers are needed if they work 8 hours daily.

SOLUTION

Let x be the number of workers to make 720 units in 12 days working 8 hours daily.

Days	Hours	Units	Workers
10	10	500	50
12	8	720	x

Ratio of days = 12 : 10 (inverse)

Ratio of hours = 8 : 10 (inverse)

Ratio of units = 500 : 720 (direct)

Ratio of workers = 50 : x

In proportion

$$\left.\begin{matrix}12:10\\8:10\\500:720\end{matrix}\right\} :: 50:x$$

Applying the fundamental principle of proportion

$$x \times 500 \times 8 \times 12 = 50 \times 10 \times 10 \times 720$$

$$x = \frac{50 \times 10 \times 10 \times 720}{500 \times 8 \times 12} = 75 \text{ workers}$$

-:1.30:-

A fleet of 15 similar cars, uses 625 litres of petrol in 5 days. How long will 600 litres if 3 cars are laid up for repairs?

SOLUTION

Let x be the number of days to use 600 litre petrol by 12 cars.

Cars	Petrol (in litre)	Days
15	625	5
12	600	x

Ratio of men = 12 : 15 (inverse)
Ratio of hours = 625 : 600 (direct)
Ratio of days = 5 : x

In proportion

$$\begin{matrix} 12 : 15 \\ 625 : 600 \end{matrix} :: 5 : x$$

Applying the fundamental principle of proportion

$$x \times 625 \times 12 = 5 \times 15 \times 600$$

$$x = \frac{5 \times 15 \times 600}{625 \times 12} = 6 \text{ days}$$

SET – B

-:1.1:-

A and B are partners sharing profit and loss in the ratio of 3:2. A new partner C is admitted and ¼ share of profit is given. What is the new profit sharing ratio?

SOLUTION

We can find the common denominator to equivalent fractions.
When profit is 3 : 2, then share of A and B are

$$\frac{3}{5}, \frac{2}{5}$$

When partner C is admitted, then the ratio is:

$$\frac{3}{5}:\frac{2}{5}:\frac{1}{4}=\frac{3\times4}{5\times4}:\frac{2\times4}{5\times4}:\frac{1\times5}{4\times5}$$

$$=\frac{12}{20}:\frac{8}{20}:\frac{5}{20}$$

Total of ratios = 12 + 8 + 5 = 25
New profit ratio is:

$$\text{A's Share}=\frac{12}{25}$$

$$\text{B's Share}=\frac{8}{25}$$

$$\text{C's Share}=\frac{5}{25}$$

-:1.2:-

Mr. Ajmal had Rs. 42000. He owed Rs. 4000. He died and burial expenses were Rs. 2000. He left behind one widow, 3 sons and 2 daughters. The balance is to be paid in the ratio 1/8 to widow, while the share of each son will be twice as compared to daughter. Find the share of each.

SOLUTION

Total Estate	= Rs. 42000
Debt	= Rs. 4000
	Rs. 38000
Burial Expenses	= Rs. 2000
Balance	**= Rs. 36000**

$$\text{Share of widow} = 36000 \times \frac{1}{8} = \text{Rs. } 4500$$

Balance = Rs. 36000 – 4500 = Rs. 31500

Ratio of a son and a daughter is 2 : 1

There are 3 sons and 2 daughters

So $2 + 2 + 2 + 1 + 1 = 8$ shares

$$\text{Share of each Son} = \frac{2}{8} \times 31500 = \text{Rs. } 7875$$

$$\text{Share of each Daughter} = \frac{1}{8} \times 31500 = \text{Rs. } 3937.50$$

-:1.3:-

As estate of Rs. 168000 is to be divided among the heirs in the ratio of 1/4: 1/3 : 2/5 : 5/12. What is the share of each member.

SOLUTION

The ratio of shares is

$$\frac{1}{4} : \frac{1}{3} : \frac{2}{5} : \frac{5}{12}$$

We can find the common denominator to equivalent fractions:

$$\frac{1}{4} : \frac{1}{3} : \frac{2}{5} : \frac{5}{12} = \frac{1 \times 15}{4 \times 15} : \frac{1 \times 20}{3 \times 20} : \frac{2 \times 12}{5 \times 12} : \frac{5 \times 5}{12 \times 5}$$

$$= \frac{15}{60} : \frac{20}{60} : \frac{24}{60} : \frac{25}{60}$$

Ratio = 15 : 20 : 24 : 25

Sum of ratio = 15 + 20 + 24 + 25 = 84

$$\text{Frist Share} = \frac{15}{84} \times 168000 = \text{Rs. } 30000$$

$$\text{Second Share} = \frac{20}{84} \times 168000 = \text{Rs. } 40000$$

$$\text{Third Share} = \frac{24}{84} \times 168000 = \text{Rs. } 48000$$

$$\text{Fourth Share} = \frac{25}{84} \times 168000 = \text{Rs. } 50000$$

-:1.4:-

A contractor agreed to mix gravel, sand and cement in the ratio 8:5:3 in concrete work. He used 48000 lbs of mixture in completion of work. What is the quantity of each material used?

SOLUTION

The ratio of Gravel, Sand and Cement is 8 : 5 : 3

Sum of ratio = 8 + 5 + 3 = 16

Total mixture = 48000 lbs

$$\text{Gravel} = \frac{8}{16} \times 48000 = 24000 \text{ lbs}$$

$$\text{Sand} = \frac{5}{16} \times 48000 = 15000 \text{ lbs}$$

$$\text{Cement} = \frac{3}{16} \times 48000 = 9000 \text{ lbs}$$

-:1.5:-

Three milkmen mixes milk with water in the following ratios:

First milkman's ratio = 7 : 1

Second milkman's ratio = 9 : 2

Third milkman's ratio = 10 : 1

Who is more dishonest out of three milkmen?

SOLUTION

Here three ratios are

$$7:1 = \frac{1}{7}; \qquad 9:2 = \frac{2}{9}; \qquad 10:1 = \frac{1}{10}$$

$$\frac{1}{7}:\frac{2}{9}:\frac{1}{10}$$

We can find the common denominator to equivalent fractions

$$\frac{1\times 90}{7\times 90}:\frac{2\times 70}{9\times 70}:\frac{1\times 63}{10\times 63} = \frac{90}{630}:\frac{140}{630}:\frac{63}{630}$$

Second milkman is more dishonest as his ratio is greater.

-:1.6:-

A provides capital of Rs. 10000 for one year. B provides capital of Rs. 15000 for 8 months and C Rs. 20000 for 3 months. A profit of Rs. 37500 is to be distributed in the ratio of capital.

SOLUTION

Here total profit = Rs. 37500

The ratio of investment of A, B and C is

$$10000 : 15000 : 20000$$

The ratio of time is months of investment is

$$12 : 8 : 3$$

So the ratio of investment with respect to time is

$$10000 \times 12 : 15000 \times 8 : 20000 \times 3$$

$$10 \times 12 : 15 \times 8 : 20 \times 3$$

$$120 : 120 : 60$$

$$2 : 2 : 1$$

Sum of ratios = 2 + 2 + 1 = 5

$$\text{Share of A's Profit} = \frac{2}{5} \times 37500 = \text{Rs. } 15000$$

$$\text{Share of B's Profit} = \frac{2}{5} \times 37500 = \text{Rs. } 15000$$

$$\text{Share of C's Profit} = \frac{1}{5} \times 37500 = \text{Rs. } 7500$$

-:1.7:-

The profit sharing ratio between A and B is 2 : 3. The ratio between B and C is 4 : 7. The firm earned a profit of Rs. 41000. What is the amount of each partner?

SOLUTION

It is the problem of continued ratio.

Total Profit = Rs. 41000

$$\begin{array}{ccccc} A & : & B & : & C \\ 2 & : & 3 & & \\ & & 4 & : & 7 \\ \hline 8 & : & 12 & : & 21 \end{array}$$

Sum of Ratios = 8 + 12 + 21 = 41

$$\text{Share of A's Profit} = \frac{8}{41} \times 41000 = \text{Rs. } 8000$$

$$\text{Share of B's Profit} = \frac{12}{41} \times 41000 = \text{Rs. } 12000$$

$$\text{Share of C's Profit} = \frac{21}{41} \times 41000 = \text{Rs. } 21000$$

-:1.8:-

A storekeeper can keep 720 cans of vegetables. The owner wants to buy peas, beans and corn in the ratio of 3 : 4 : 5. What is the size of order of each kind.

SOLUTION

Total Cans = 720 cans

Ratio of Vegetables cans

Peas : Beans : Corn = 3 : 4 : 5

Sum of ratios = 3 + 4 + 5 = 12

$$\text{Order of Peas} = \frac{3}{12} \times 720 = 180 \text{ cans}$$

$$\text{Order of Beans} = \frac{4}{12} \times 720 = 240 \text{ cans}$$

$$\text{Order of Corn} = \frac{5}{12} \times 720 = 300 \text{ cans}$$

-:1.9:-

A farmer has 840 acres of land for planting rice, wheat, cotton and fodder in the ratio of 5 : 4 : 3 : 2 respectively. What are acres used for each crop.

SOLUTION

Total Land = 840 acres

Ratio of Rice, Wheat, Cotton and Fodder is

Rice : Wheat : Cotton : Fodder

5 : 4 : 3 : 2

Sum of ratios = 5 + 4 + 3 + 2 = 14

$$\text{Land used for Rice} = \frac{5}{14} \times 840 = 300 \text{ acres}$$

$$\text{Land used for Wheat} = \frac{4}{14} \times 840 = 240 \text{ acres}$$

$$\text{Land used for Cotton} = \frac{3}{14} \times 840 = 180 \text{ acres}$$

$$\text{Land used for Fodder} = \frac{2}{14} \times 840 = 120 \text{ acres}$$

-:1.10:-

A milk man mixes milk with water in the ratio 7 : 2 respectively. He has 81 litre of mixed milk. What is the quantity of pure milk?

SOLUTION

Total mixed Milk = 81 litre

Ratio of Milk to Water is

Milk : Water = 7 : 2

Sum of ratios = 7 + 2 = 9

$$\text{Amount of Pure Milk} = \frac{7}{9} \times 81 = 63 \text{ litre}$$

-:1.11:-

The property of a person is valued Rs. 330000. He has two wives, one son and 4 daughters. The share of wife is 1/8 each. The share of a son is double than share of each daughter. Distribute the property.

SOLUTION

Total value of property = Rs. 3,30,000

He has 2 Wives, 1 son and 4 daughter

$$\text{Share of each Wife} = \frac{1}{8} \times 330000 = \text{Rs. } 41250$$

Share of both Wives = Rs. 41250 + Rs. 41250 = Rs. 82500

Balance = Total – Wives' share

= Rs. 330000 – Rs. 82500 = Rs. 247500

Shares of Son and Daughters

2 : 1 : 1 : 1 : 1

Sum of Ratios = 2 + 1 + 1 + 1 +1 = 6

$$\text{Share of Son} = \frac{2}{6} \times 247500 = \text{Rs. } 82500$$

$$\text{Share of each Daughter} = \frac{1}{6} \times 247500 = \text{Rs. } 41250$$

-:1.12:-

The estate of a person consists of 400 cows and 1200 goats. He had a widow, 3 sons and 4 daughters. The claim of widow is 1/8. The share of a son is double than a daughter. Distribute the cows and goats among them.

SOLUTION

Total Number of Cows = 400

Total Number of Goats = 1200

$$\text{Cow's Share of Widow} = \frac{1}{8} \times 400 = 50 \text{ cows}$$

$$\text{Goats Share of Widow} = \frac{1}{8} \times 1200 = 150 \text{ goats}$$

Remaining Cows = 350 & Remaining Goats = 1050

He has 3 Sons and 4 Daughters. Ratio is

$$2 : 2 : 2 : 1 : 1 : 1 : 1$$

Sum of Ratios = 2 + 2 + 2 + 1 + 1 + 1 + 1 = 10

$$\text{Cow's Share of each Son} = \frac{2}{10} \times 350 = 70 \text{ cows}$$

$$\text{Cow's Share of each Daughter} = \frac{1}{10} \times 350 = 35 \text{ cows}$$

$$\text{Goat's Share of each Son} = \frac{2}{10} \times 1050 = 210 \text{ goats}$$

$$\text{Goat's Share of each Daughter} = \frac{1}{10} \times 1050 = 105 \text{ goats}$$

-:1.13:-

The tool expenses of a factory are Rs. 2552. The machine hours of two departments A and B are 1800 and 1500 respectively. Distribute the tool expenses on basis of machine hours.

SOLUTION

Total Expenses = Rs. 2550

Ratios of Machine hours of A and B Departments

$$A : B = 1800 : 1500$$
$$18 : 15$$
$$6 : 5$$

Sum of Ratios = 6 + 5 = 11

$$\text{Expenses of A Department} = \frac{6}{11} \times 2552 = \text{Rs.}1392$$

$$\text{Expenses of B Department} = \frac{5}{11} \times 2552 = \text{Rs.}1160$$

-:1.14:-

Javaid is half as old as Rashid and Rashid is half as old as Naveed. The sum of their ages is 105 years. Calculate the ages of Javaid, Rashid and Naveed.

SOLUTION

Ratios of ages are as

Javaid is half as old as Rashid i.e. Rashid is double as old as Javaid

So the ratio is 1 : 2

Similarly Rashid is half as old as Naveed. that is Naveed is double as old as Rashid

So, ratio is 1 : 2

Hence

Javaid :	Rashid :	Naveed
1 :	2	
	1 :	2
1 :	2 :	4

Sum of Ratios = 1 + 2 + 4 = 7

$$\text{Javaid's Age} = \frac{1}{7} \times 105 = 15 \text{ years}$$

$$\text{Rashid's Age} = \frac{2}{7} \times 105 = 30 \text{ years}$$

$$\text{Naveed's Age} = \frac{4}{7} \times 105 = 60 \text{ years}$$

-:1.15:-

A purchase manager paid carriage of rupees for A, B and C materials. The weight ratio is A : B = 4 : 7 and B : C = 6 : 3. What is carriage for each material.

SOLUTION

Total Carriage Bill = Rs. 11725

Ratio of materials is

A	:	B	:	C
4	:	7		
		6	:	3
4	:	42	:	21

Sum of Ratios = 4 + 42 + 21 = 67

$$\text{Carriage Share of Material A} = \frac{4}{67} \times 11725 = \text{Rs. } 700$$

$$\text{Carriage Share of Material B} = \frac{42}{67} \times 11725 = \text{Rs. } 7350$$

$$\text{Carriage Share of Material C} = \frac{21}{67} \times 11725 = \text{Rs. } 3675$$

-:1.16:-

The cost of 20 lbs of tea is Rs. 1700. What should be the cost of 33 lbs of tea?

SOLUTION

Tea (in lbs)	Cost (in Rs.)
20	1700
33	x

20 : 33 :: 1700 : x (Direct proportion)

Applying the fundamental principal of proportion

$$20 \times x = 1700 \times 33$$

$$x = \frac{1700 \times 33}{20} = \text{Rs. } 2805$$

-:1.17:-

18 men complete the job in 30 days. How long it will take to complete the job by 12 workers?

SOLUTION

Workers	Days
18	30
12	x

12 : 18 :: 30 : x (Inverse proportion)

Applying the fundamental principal of proportion

$$12 \times x = 30 \times 18$$

$$x = \frac{30 \times 18}{12} = 45 \text{ days}$$

-:1.18:-

The cost of 5 kilograms of mutton is Rs. 1250. Calculate the cost of 43 kilograms of mutton when rate remains the same.

SOLUTION

Let x be the cost of 43 kgs of mutton.

In this problem estimated value (cost) will be greater. so we form proportion with first ratio as smaller.

No of Kg	Cost (in Rs.)
5	1250
43	x

$$\frac{5}{43} = \frac{1250}{x}$$

5 : 43 :: 1250 : x (Direct proportion)

Applying the fundamental principal of proportion

$$5 \times x = 1250 \times 43$$

$$x = \frac{1250 \times 43}{5} = \text{Rs.}10750$$

-:1.19:-

The cost of 7 litres of cooking oil is Rs. 525. What is the cost of 25 litres of such oil if the rate is the same.

SOLUTION

Let x be the cost of 25 litres of oil.

In this problem estimated value (cost) will be greater, so we form proportion with first ratio as smaller.

No of Litres	**Cost (in Rs.)**
7	525
25	x

$$\frac{7}{25} = \frac{525}{x}$$

7 : 25 :: 525 : x (Direct proportion)

Applying the fundamental principal of proportion

$$7 \times x = 525 \times 25$$

$$x = \frac{525 \times 25}{7} = \text{Rs.}1875$$

-:1.20:-

Numan earns Rs. 700 by investing Rs. 20,000. How much Aslam will earn on investment of Rs. 55000 under the same scheme of investment.

SOLUTION

Let x be the earning on investment = Rs. 55000

Earning (Rs.)	**Investment (Rs.)**
700	20000
x	55000

$$\frac{700}{x} = \frac{20000}{55000}$$

700 : x :: 20000 : 55000 (Direct proportion)

Applying the fundamental principal of proportion

$$700 \times 55000 = 20000 \times x$$

$$20000x = 700 \times 55000$$

$$x = \frac{700 \times 55000}{20000} = \text{Rs.}1925$$

-:1.21:-

A factory manager plan to produce 200 units with the help of 50 workers who work 8 hours a day. How many units can be made by 80 workers if they work 6 hours per day.

SOLUTION

Let x be the units produced working 6 hours by 80 workers.

Ratio of workers = 50 : 80 (Direct proportion)

Ratio of hours = 8 : 6 (Direct proportion)

Ratio of units = 200 : x

In proportion

$$\left.\begin{matrix}50:80\\8:6\end{matrix}\right\} :: 200 : x$$

Applying the fundamental principle of proportion

$$x \times 8 \times 50 = 200 \times 80 \times 6$$

$$x = \frac{200 \times 80 \times 6}{8 \times 50} = 240 \text{ units}$$

-:1.22:-

A carton factory makes 160 boxes in 16 days with the help of 48 workers. How many days are needed to make 200 boxes with the help of 40 workers.

SOLUTION

Let x be the number of days required.

Days	Workers	Boxes
16	48	160
x	40	200

Ratio of workers = 40 : 48 (Inverse)

Ratio of boxes = 160 : 200 (Direct)

Ratio of days = 16 : x

Proportion is

$$\left.\begin{matrix}40:48\\160:200\end{matrix}\right\} :: 16 : x$$

Applying the fundamental principle of proportion

$$x \times 160 \times 40 = 16 \times 48 \times 200$$

$$x = \frac{16 \times 48 \times 200}{160 \times 40} = 24 \text{ days}$$

-:1.23:-

A labour force of 15 workers complete the job in 15 days for 8 hours a day. How many days are needed to complete it if 25 workers work for 6 ho"rs a day.

SOLUTION

Let x be the required number of days.

Days	Workers	Hours
15	15	8
x	25	6

Ratio of workers = 25 : 15 (Inverse)
Ratio of hours = 6 : 8 (Inverse)
Ratio of days = 15 : x
Proportion is

$$\begin{matrix} 25 : 15 \\ 6 : 8 \end{matrix} :: 15 : x$$

Applying the fundamental principle of proportion

$$x \times 6 \times 25 = 15 \times 15 \times 8$$

$$x = \frac{15 \times 15 \times 8}{6 \times 25} = 12 \text{ days}$$

EXERCISE NO. 2

SET – A

-:2.1:-

Express as percents

i) $\frac{1}{2}$ ii) $\frac{3}{5}$ iii) $\frac{3}{25}$ iv) $1\frac{1}{2}$

v) $2\frac{1}{4}$ vi) $\frac{1}{50}$ vii) $\frac{8}{10}$ viii) $\frac{1}{4}$

SOLUTION

(i) $\frac{1}{2}=\left(\frac{1}{2}\times 100\right)\% = 50\%$

(ii) $\frac{3}{5}=\left(\frac{3}{5}\times 100\right)\% = 60\%$

(iii) $\frac{3}{25}=\left(\frac{3}{25}\times 100\right)\% = 12\%$

(iv) $1\frac{1}{2}=\frac{3}{2}=\left(\frac{3}{2}\times 100\right)\% = 150\%$

(v) $2\frac{1}{4}=\frac{9}{4}=\left(\frac{9}{4}\times 100\right)\% = 225\%$

(vi) $\frac{1}{50}=\left(\frac{1}{50}\times 100\right)\% = 2\%$

(vii) $\frac{8}{10}=\left(\frac{8}{10}\times 100\right)\% = 80\%$

(vii) $\frac{1}{4}=\left(\frac{1}{4}\times 100\right)\% = 25\%$

-:2.2:-

Express the decimals as percents.

i) 0.25 ii) 0.06 iii) 0.89 iv) 0.17

SOLUTION

(i) 0.25 = (0.25 x 100)% = 25%

(ii) 0.06 = (0.06 x 100)% = 6%

(iii) 0.89 = (0.89 x 100)% = 89%

(iv) 0.17 = (0.17 x 100)% = 17%

-:2.3:-

Express following percents as fractions.

i) 50% **ii) 25%** **iii) 16%** **iv) 230%**
v) 87½% **vi) 20%** **vii) 4%** **viii) 99%**

SOLUTION

(i) $50\% = \frac{50}{100} = \frac{1}{2}$

(ii) $25\% = \frac{25}{100} = \frac{1}{4}$

(iii) $16\% = \frac{16}{100} = \frac{4}{25}$

(iv) $230\% = \frac{230}{100} = \frac{23}{10}$

(v) $87\frac{1}{2}\% = \frac{175}{2}\% = \frac{175}{100} = \frac{7}{8}$

(vi) $20\% = \frac{20}{100} = \frac{1}{5}$

(vii) $4\% = \frac{4}{100} = \frac{1}{25}$

(vii) $99\% = \frac{99}{100}$

-:2.4:-

Express these percents as decimals.

i) 15% **ii) 85%** **iii) 12½%** **iv) 6¼%**

SOLUTION

(i) $15\% = \frac{15}{100} = 0.15$

(ii) $85\% = \frac{85}{100} = 0.85$

(iii) $12\frac{1}{2}\% = \frac{25}{2}\% = \frac{25}{2 \times 100} = 0.125$

(iv) $6\frac{1}{4}\% = \frac{25}{4}\% = \frac{25}{4 \times 100} = 0.625$

-:2.5:-

Find a) 5% of 200 b) 6% of 180 c) 12% of 15
d) 23% of 4500 e) 85% of 400 f) ¾% of 2000

SOLUTION

(a) 5% is the rate and 200 is the base.

Thus percentage = Base x Rate%

$$= \frac{200 \times 5}{100} = 10$$

(b) 6% is the rate and 180 is the base.

Thus percentage = Base x Rate%

$$= \frac{\text{Base} \times \text{Rate}}{100} = \frac{180 \times 6}{100} = 10.8$$

c) 12% is the rate and 15 is the base

Thus percentage = Base x Rate%

$$= \frac{\text{Base} \times \text{Rate}}{100} = \frac{15 \times 12}{100} = \frac{180}{100} = 1.80$$

(d) 23% is the rate and 4500 is the base

Thus percentage = Base x Rate%

$$= \frac{\text{Base} \times \text{Rate}}{100} = \frac{4500 \times 23}{100} = 1035$$

(e) 85% is the rate and 400 is the base.

Thus percentage = Base x Rate%

$$= \frac{\text{Base} \times \text{Rate}}{100} = \frac{400 \times 85}{100} = 340$$

(f) ¾% is the rate and 2000 is the base.

Thus percentage = Base x Rate%

$$= \frac{\text{Base} \times \text{Rate}}{100} = \frac{2000 \times 3}{100 \times 4} = 15$$

-:2.6:-

If 5% of a number is 60, what is the number?

SOLUTION

$$\text{Number} = \frac{\text{Percentage of the Number}}{\text{Rate\%}}$$

$$\text{Number} = \frac{\text{Percentage of the Number} \times 100}{\text{Rate}}$$

Here percentage of number = 60, Rate % = 5% = 0.05

$$\text{Number} = \frac{60 \times 100}{5} = 1200$$

-:2.7:-

If ½% of a number is 82.46, what is the number?

SOLUTION

$$\text{Number} = \frac{\text{Percentage of the Number} \times 100}{\text{Rate}}$$

Percentage of the number = 82.46

Rate = ½%

$$\text{Number} = \frac{82.46 \times 100}{\frac{1}{2}} = 164.92 \times 100 = 16492$$

-:2.8:-

If 45% of number is 3000, what is the number?

SOLUTION

Here percentage of a number = 300; Rate% = 45%

$$\text{Number} = \frac{\text{Percentage of the Number} \times 100}{\text{Rate}}$$

$$= \frac{3000 \times 100}{45} = \frac{20000}{3} = 6666.67$$

-:2.9:-

Mr. Khalid bought a radio for Rs. 1200 and he sold it for Rs. 1425. Find his percent profit.

SOLUTION

Selling price of radio = Rs. 1425

Purchased price = Rs. 1200

Profit = Selling Price - Purchase price

= Rs. 1425 - Rs. 1200= Rs. 225

Here Rs. 225 is the number for which percent (Rate) is required on the base of Rs. 1200.

Percent profit = Rate%

$$\text{Rate\%} = \frac{\text{Number}}{\text{Base}} = \frac{225}{1200} = 18\frac{3}{4}\%$$

-:2.10:-

Mr. Noor Muhammad gets 15% profit from his investment. He gets Rs. 400 as net profit, find his investment.

SOLUTION

Rate % = 15%

Percentage of profit = Rs. 400

Investment = Amount

$$\text{Amount of Investment} = \frac{\text{Percentage of Profit}}{\text{Rate\%}}$$

$$= \frac{\text{Rs. } 400 \times 100}{15} = \frac{8000}{3} = 2666.67$$

-:2.11:-

Of the 80,000 seats in the football stadium 48640 were filled. What percent of the stadium was filled.

SOLUTION

Here 48640 is the number for which percent (rate) is required on the base 80000.

$$\text{Percent} = \text{Rate \%} = \frac{\text{Number}}{\text{Base}} = \frac{48640}{80000} = 60.8\%$$

-:2.12:-

If your automobile payments are Rs 300, a month, what percent of year Rs. 5000 per month salary must be set a side to pay for your automobile.

SOLUTION

Here 300 is the number for (rate) is required on the base 5000.

$$\text{Percent} = \text{Rate \%} = \frac{\text{Number}}{\text{Base}} = \frac{300}{5000} = \frac{3}{50} = 6\%$$

-:2.13:-

About 70% of wheat can be converted into flour. To the nearest kg, how many kg of wheat are needed to make 100 kg of flour.

SOLUTION

Here rate % of wheat converted into flour is 70%

Percentage = 100 kg

We have to find to total wheat i.e. Base

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate}\ \%} = \frac{\text{Percentage} \times 100}{\text{Rate}}$$

$$= \frac{100 \times 100}{70} = \frac{10000}{70} = 142.857 = 143 \text{ kg}$$

-:2.14:-

A man paid zakat of Rs. 312.50 at the rate of 2½ % p.a. of his wealth. What is the value of his wealth?

SOLUTION

Here total zakat paid = Percentage = Rs. 312.50

Rate = 2½ % = $^5/_2$%

We have to find his wealth i.e. Base

$$\text{Total Wealth} = \text{Base} = \frac{\text{Percentage}}{\text{Rate}\ \%}$$

$$= \frac{\text{Percentage} \times 100}{\text{Rate}} = \frac{312.50 \times 100 \times 2}{5} = \text{Rs. } 12500$$

-:2.15:-

The selling price of a steal cabinet is Rs. 840, and the gross profit is 10% of the cost. Find the cost.

SOLUTION

The cost is the base from which gross profit of 10% has been calculated.

The given selling price = Cost + Gross profit

= 100 + 10 = 110% of Cost

Since Rs. 840 is 110% of the cost. the Cost is

$$\text{Cost} = \frac{840}{110\%} = \frac{840 \times 100}{110} = \text{Rs. } 763.63$$

-:2.16:-

Find the selling price of typewriter which costs Rs. 4000 and the mark up is 8%.

SOLUTION

Here cost of type writer = C = 4000: Mark up percent = R = 8%

We have

$$S = C(1+R)$$
$$S = 4000(1 + 0.08)$$
$$S = 4000(1.08)$$
$$S = Rs.\ 4320.00$$

-:2.17:-

A man sold a fountain pen for Rs. 125. his profit was 30% of his original purchase price. What was his purchase price?

SOLUTION

Here selling price of fountain pen =S= Rs. 125

Rate of profit = 30%

Purchase price = Cost price = C

$$S = C(1+R)$$
$$125 = C(1 + 0.30)$$
$$125 = 1.30\ C$$
$$Cost = \frac{125}{130} = Rs.\ 96.15$$

So price of fountain pen is Rs. 96.15

-:2.18:-

What is the cost of coloured SONY TV. set which is sold for Rs. 9800, if the percent mark up is 40% on cost.

SOLUTION

We have $S = C(1 + R)$

Here $S = Rs.\ 9800$ and

Percent mark up = R = 40%

$9800 = C(1 + 0.40)$

$9800 = 1.40C$

$$Cost = \frac{9800}{1.40} = \frac{980000}{140} = Rs.\ 7000$$

-:2.19:-

After a mark up of 25% on sales a watch is sold for Rs. 1000.

i) What is its cost price.

ii) What is the percent mark up on sales if the cost price of watch have been Rs. 700.

SOLUTION

Here

Mark up percent on sale P = 25%= 0.25

Selling price of watch = S = Rs. 1000

(i) We know that

Cost =C= S(1 - P)

C = 1000(1-025)

C = 1000(0.75)

C = Rs. 750

(ii) Cost of watch = C = Rs. 700

Selling price = S = Rs. 1000

C = S(1 - P)

700 = 1000(1 - P)

700 = 1000 - 1000P

1000P = 1000 - 700

1000P = 300

P = 0.30

P = 30 %

Hence mark up on sale = 30%

-:2.20:-

A and B together invest Rs. 52000 in a small business. A's share of investment is 36% of this amount. Find out B's investment in this business.

SOLUTION

Total investment = Rs. 52000

Let the total investment = Rs. 100

A's share of investment = Rs. 36 = 36%

B's share of investment = Rs. 64 = 64%

B's amount of investment = Total investment x Rate %

$$= \frac{\text{Total Investment} \times \text{Rate}}{100}$$

$$= \frac{52000 \times 64}{100} = \text{Rs. } 33280$$

-:2.21:-

Last month a store's sales were Rs. 25000 and this month the sales are Rs. 45230. Find out the percentage increase in sales.

SOLUTION

Last month's sale = Rs. 25000

Sale of this month = Rs. 45230

Increase = Rs. 45230 - Rs. 25000

= Rs. 20230

$$\text{Percentage Increase} = \frac{20230 \times 100}{2500} = 80.92\%$$

-:2.22:-

Sultan Ahmad purchased merchandise for Rs. 7500. He was allowed a trade discount of 7%. Find the amount of discount.

SOLUTION

Here Total price = Rs. 7500

Rate of discount = 7%

$$\text{Amount of Discount} = \frac{\text{Total Price} \times \text{Rate of Discount}}{100}$$

$$= \frac{7500 \times 7}{100} = \text{Rs. } 525$$

-:2.23:-

Mr. Naveed purchase 50 books of Economics from Kitab Markaz Faisalabad. The printed price of each book is Rs. 35.00. if a trade discount of 15% is allowed to him find the discount and net price of the books.

SOLUTION

Printed price of a book = Rs. 35.00

Printed price of 50 books = Rs. 1750.00

Trade discount = 15%

$$\text{Amount of Discount} = \frac{\text{Total Price} \times \text{Rate of Discount}}{100}$$

$$= \frac{1750 \times 15}{100} = \frac{26250}{100} = \text{Rs. } 262.50$$

Net price of the book = Price-Total discount
= Rs. 1750.00 - Rs. 262.50
= Rs. 1487.50

-:2.24:-

A man paid Rs. 555 for a radio set after 7½% discount had been deducted. What was the market price of the set.

SOLUTION

Net price = Rs. 555

Rate of discount = 7½%

Let the marked price of radio = Rs. 100

Discount = Rs. 7.50

Net price of radio = Rs. 92.50

If net price is Rs. 92.50 the market price =100

If net price is Rs. 555 the market price =

$$\frac{100 \times 555}{92.50} = \frac{10000 \times 555}{92.50} = \text{Rs. } 600$$

-:2.25:-

On the purchase of a car the dealer allowed to a customer discount at the rate of 4½%. Find out the total price of a car and amount paid to dealer, if the discount allowed was Rs. 6000.

SOLUTION

Discount allowed = Rs. 6000

Rate of discount = 4½%

Total price of car

$$\text{Total Price} = \frac{\text{Discount} \times 100}{\text{Rate of Discount}}$$

$$= \frac{6000 \times 100 \times 2}{9} = \text{Rs. } 133333.33$$

Total Discount = 6000

Amount paid to the Dealer = Rs. 133333.33 – Rs. 6000
= Rs. 127333.33

-:2.26:-

A T-shirt marked at Rs. 145 was sold for Rs. 100 in a clearance sale. Find the rate of discount.

SOLUTION

Marked price of T-Shrit = Rs. 145

Selling price = Rs. 100

Discount = Rs. 45

$$\text{Rate of Discount} = \frac{\text{Discount} \times 100}{\text{Price}}$$

$$= \frac{45 \times 100}{145} = 31\frac{1}{29}\%$$

-:2.27:-

A furniture dealer paid Rs. 10,000 for 50 lawn chairs on which he had been given discount of 12%. Find the price of a lawn chair.

SOLUTION

Net price of 50 lawn chairs = Rs. 10,000

Net price of one lawn chair = Rs. 200

Let the list price of a lawn chair = Rs. 100

Discount at 12% = Rs. 12

Net price = Rs. 88

If net price is Rs. 88, marked price = Rs. 100

If net price is Rs. 200, marked price

$$= \frac{100 \times 200}{88\ \cancel{99}} = \frac{2500}{11} = \text{Rs. } 227.27$$

-:2.28:-

Mr. Zahoor allows 10% trade discount and 3% cash discount. Find the net amount be paid, if the marked price of a cooler is Rs. 13500.

SOLUTION

Marked price = Rs. 13500

Less 10% trade discount on Rs. 13500 = Rs. 1350

3% cash discount as Rs. 12150 = Rs. 364.50

Total trade and cash discount

= Rs. 1350 + Rs. 364.50 = Rs. 1740.50

Net amount = Rs. 13500 - Rs. 1714.50 =Rs.11785.50

-:2.29:-

A house was sold for Rs. 78,350. The property dealer was allowed a commission of 2%. Find out his commission.

SOLUTION

Total price of house = Rs. 78350

Rate of commission = 2%

$$\text{Commission} = \frac{\text{Total Price} \times \text{Rate}}{100}$$

$$= \frac{78350 \times 2}{100} = \text{Rs. } 1567$$

-:2.30:-

A widow has rented out her property at Rs. 4500 pm. An income of Rs. 30000 pa is tax exempted. Calculate property tax on the balance of here income at a rate of 10%.

SOLUTION

The per month rent = Rs. 4500

Income of a year = 4500 x 12 = Rs. 54000

Exempted income = Rs. 30000

Balance = Rs. 24000

Property tax rate = 10%

Here Base = Rs. 24000

$$\text{Tax} = \text{Base} \times \text{Rate}\,\% = \frac{\text{Base} \times \text{Rate}}{100}$$

$$= \frac{24000 \times 10}{100} = \text{Rs. } 2400$$

-:2.31:-

A salesman is paid a salary of Rs. 500 a month and 1% commission on sales. If his total income in one month is Rs. 750, find the value of his sales in that month.

SOLUTION

Total income of Saleman = Rs. 750

Salary = Rs. 500

Amount of commission = Rs. 250

$$\text{Sales for the Month} = \frac{250 \times 100}{1} = \text{Rs. } 25000$$

-:2.32:-

At 5% rate of commission, Miss Nelson, a sales girl got Rs. 250 on the sale of talcum face powder. Find the value of her sales. If the price of face powder was Rs. 2.50 per container, how many containers did she sell?

SOLUTION

Miss Nelson get Rs. 5 when the sale is for Rs. 100. She gets total commission Rs. 250.

$$\text{Total Sale} = \frac{250 \times 100}{5} = \text{Rs. } 5000$$

Price of a container = Rs. 2.50

$$\text{The number of container she sells} = \frac{5000}{2.50} = 2000 \text{ Containers}$$

-:2.33:-

Find the total amount of commission on sale to Rs. 8000, if the commission rate is 8%.

SOLUTION

Total sale = Rs. 8000

Rate of commission = 8%

$$\text{Total amount of Commission} = \frac{\text{Total Sale} \times \text{Rate}}{100}$$

$$= \frac{8000 \times 8}{100} = \text{Rs. } 640$$

SET – B

-:2.1:-

Jamal's salary is subject to 16% payroll deduction. Calculae his take home pay for a month if his monthly salary is Rs. 8500.

SOLUTION

Jamal's Monthly Pay = Rs. 8500

Payroll Deduction = 16%

Here Base = Rs. 8500 & Rate% = 16%

We are looking to find take home pay, that is pay after payroll deduction.

$$\text{Deduction} = \text{Percentage} = \text{Base} \times \text{Rate\%}$$
$$= 8500 \times 16\%$$
$$= 8500 \times \frac{16}{100} = 8500 \times 0.16 = \text{Rs. } 1360$$

Hence Take Home Pay = Rs. 8500 – Rs. 1360 = Rs. 7140

-:2.2:-

Ahmad plans to buy a house that costs Rs. 450,000. If he must make a down payment of 15%, how much will the down payment be?

SOLUTION

Here Cost of House = Base = Rs. 4,50,000

Rate% = Rate of down Payment = 15%

$$\text{Amount of Down Payment} = \text{Percentage} = \text{Base} \times \text{Rate\%}$$
$$= 450000 \times 15\%$$
$$= 450000 \times \frac{15}{100} = 450000 \times 0.15$$
$$= \text{Rs. } 67500$$

-:2.3:-

Zahid plans to buy a new car that costs Rs. 6,50,000. The dealer requires a down payment of 20%. How much will the down payment be?

SOLUTION

Cost of New Car = Base = Rs. 6,50,000

Rate of Down Payment = Rate% = 20%

Amount of Down Payment = Percentage = Base × Rate%

$= 650000 \times 20\%$

$= 650000 \times \frac{20}{100} = 650000 \times 0.20$

$= \text{Rs.}1,30,000$

-:2.4:-

Monica wants to buy a new refrigerator that cost Rs. 12500. She must pay 12% down payment. How much she will pay?

SOLUTION

Cost of New Refrigerator = Base = Rs. 12,500

Rate of Down Payment = Rate% = 12%

Amount of Down Payment = Percentage = Base × Rate%

$= 12500 \times 12\%$

$= 12500 \times \frac{12}{100} = 12500 \times 0.12$

$= \text{Rs.}1500$

-:2.5:-

It is estimated that 5% fruit will be damaged during transportation. In a consignment of 7200 pounds how many pounds of usable fruit are there?

SOLUTION

Total Fruit of Consignment = Base = 7200 lbs

Rate of Damaged Fruit = Rate% = 5%

Amount of Damaged Fruit = Percentage = Base × Rate%

$= 7200 \times 5\%$

$= 7200 \times \frac{5}{100} = 7200 \times 0.05 = 360$ lbs

Hence The Amount of Fruit Usable = Total Fruit – Damaged Fruit

$= 7200 - 360 = 6840$ lbs

OR

We know that Total Fruit = 7200 lbs

If 5% is damaged fruit then usable fruit is 95%

Now Here Base = 7200 lbs & Rate% = 95%

Total Usable Fruit = Base × Rate%

$$= 7200 \times 95\%$$

$$= 7200 \times \frac{95}{100} = 7200 \times 0.95$$

$$= 6840 \text{ lbs}$$

-:2.6:-

A whole seller allows customers 2% discount for payment within 30 days. How much cash will be received from customer's bill of Rs. 5092 paid within 25 days.

SOLUTION

Rate of Discount = 2% & Base = Rs. 5092

Amount of Discount = Base × Rate%

$$= 5092 \times 2\%$$

$$= 5092 \times \frac{2}{100} = 5092 \times 0.02$$

$$= \text{Rs.}101.84$$

Cash Received from Customer = Total Bill - Discount

= Rs. 5092 – 101.84 = Rs. 4990.16

-:2.7:-

A dress company determines the selling price of its dresses by adding 35% to the cost. Calculate the selling price of a garment that cost Rs. 435.

SOLUTION

Cost Price of Garment = Base = Rs. 435

Rate of Increase = Rate% = 35%

Selling Price = Cost Price + Increase

Increase = Percentage = Base × Rate%

$$= 435 \times 35\%$$

$$= 435 \times \frac{35}{100} = 435 \times 0.35 = \text{Rs.}152.25$$

Hence Selling Price = Rs. 435 + Rs. 152.25 = Rs. 587.25

OR

If we want 35% increase then selling price will be 135% of cost We can calculate as follows:

Base = Rs. 435 & Rate% = 135%

Hence

$$\text{Seling Price} = \text{Base} \times \text{Rate\%}$$
$$= 435 \times 135\%$$
$$= 435 \times \frac{135}{100} = 435 \times 1.35 = \text{Rs.}587.25$$

-:2.8:-

A batch of goods was damaged during production. These goods were sold at 43% below cost. What was the selling price if the cost was Rs. 948790?

SOLUTION

Here Cost = Base = Rs. 9,48,790

Reduced Rate% due to Damaged Goods = 43%

$$\text{Amount of Deduction} = \text{Percentage} = \text{Base} \times \text{Rate\%}$$
$$= 948790 \times 43\%$$
$$= 948790 \times \frac{43}{100} = 948790 \times 0.43$$
$$= \text{Rs.}540810.30$$

Selling Price = Total Cost – Amount of Deduction

= Rs. 948790 – Rs. 407979.70 = Rs. 540810.30

OR

As Reduced Rate% is 43%, so Rate of Selling Price = 57% of Cost

Hence

$$\text{Selling Price} = \text{Base} \times \text{Rate\%}$$
$$= 948790 \times 57\%$$
$$\text{Selling Price} = 948790 \times \frac{57}{100} = 948790 \times 0.57$$
$$= \text{Rs.}540810.30$$

-:2.9:-

A telephone bill reads as follows:

Line Rent .... Rs. 174

Local call charges @ Rs. 2 per unit for 73 units......................Rs.
Sales tax @ 14% of local call charges ....Rs.
With holding tax @ 2.7% of local call charges........................Rs.
Calculate the bill.

SOLUTION

Here

Line Rent	Rs. 174.00
Local Call Charges @2 per unit for 73 Units	Rs. 176.00
Sale Tax @14% of local call charges	Rs. 24.64
W. H. Tax @2.7% of local call charges	Rs. 4.75
Total Bill	**Rs. 379.39**

-:2.10:-

IESCO electricity consumer bill reads as follows:

The rate of first 100 units consumed is Rs. 2.474 and rate of units over and above 100 units is Rs. 3.371. The total number of units consumed is 221.

Total cost of electricity Rs.
G.S.T @ 15% of total cost Rs.
PTV fee Rs. 25 Rs.
Current Bill Rs.
Less subsidy @ 14.52 on current bill Rs.
Bill payable within due date Rs.
Late payment surcharge @ 10% on bill payable Rs.
G.S.T on late payment surcharge @ 15% Rs.
Bill payable after due date Rs.
Calculate bill payable within due date and bill payable after due date.

SOLUTION

First of all do the bill calculations as follows:

For first 100 units @ 2.474 per unit	Rs. 247.40
For remaining 121 units @ 3.371 per unit	Rs. 407.80
Total Cost of Electricity	Rs. 655.29
G.S.T @ 15% of Total Cost	Rs. 98.29
P.T.V. Fee Rs. 25	Rs. 25.00
Current Bill	Rs. 778.58
Less Subsidy @ 14.52% on Current Bill	Rs. 130.05
Bill Payable within due Date	**Rs. 648.53**

Late Payment Surcharge @10% on Bill Payable ------- Rs. 64.85
G.S.T. on Late Payment Surcharge @ 15% --------------- Rs. 9.73
Bill Payable after due Date --------------------------- Rs. 723.11

-:2.11:-

Of the gross sales of Rs. 4,26,529, 12% were returned. Calculate the next sales.

SOLUTION

Here Total Sale = Base = Rs. 4,26,529
Returned = Rate% = 12%
Total Amount Returned = Percentage = Base × Rate%

$$= 426529 \times 12\%$$

$$= 426529 \times \frac{12}{100} = 426529 \times 0.12$$

$$= \text{Rs.}\,51183.48$$

Net Sales = Total Sales – Returned
= Rs. 426529 – Rs. 51183.48 = Rs. 375345.52

OR

When 12% returned, then net sales is 88%
Here Base = Rs. 426529 & Rate% = 88%
Net Sales = Base × Rate%

$$= 426529 \times 88\%$$

$$= 426529 \times \frac{88}{100} = 426529 \times 0.88 = \text{Rs.}\,375345.52$$

-:2.12:-

Afzal setout 255 questions and got 204 back. What percent of the questionnaires was returned?

SOLUTION

Here Base = 255 questions & Percentage = 204
We are looking to find Rate%

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{204}{255} = 0.8 = 80\%$$

-:2.13:-

Ali set 120 eggs to hatch, and 114 hatched out. What percent of egg hatched out?

SOLUTION

Here Base = 120 eggs; Percentage = 114 eggs & Rate% = ?

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{114}{120} = 0.95 = 95\%$$

-:2.14:-

Sadaf bought 288 ball points pens and 18 of them would not write. What percent of the pen was faulty?

SOLUTION

Total Ball Point Pen = Base = 288

Faulty Pen = Percentage = 18

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{18}{288} = 0.0625 = 6.25\%$$

So 6.25% pens are faulty.

-:2.15:-

A worker received wages of Rs. 2500 in the month of June. He received wages of Rs. 2700 in the month of July. What is the rate of increase?

SOLUTION

The Wages in the month of June = Rs. 2500

The Wages in the month of July = Rs. 2700

Total Increase = Rs. 2700 – Rs. 2500 = Rs. 200

Here Base = Rs. 2500; Percentage = Rs. 200 & Rate% = ?

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{200}{2500} = 0.08 = 8\%$$

Hence Wages Increase by 8%

-:2.16:-

A producer raised 20000 bales of cotton last year. He raised 24000 bales for this year. Calculate the percent increase in production.

SOLUTION

Here Last Year's Production = 20,000 bales

This Year's Production = 24,000 bales

Increase in Production – 4000 bales

Here Base = 20000; Percentage = 4000 & Rate% = ?

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{4000}{20000} = 0.20 = 20\%$$

-:2.17:-

A whole sale fruit dealer bought a carload of mangoes weighing 2758 kgs. After shorting, it was discovered that 217 kgs were spoiled. Find the percent of spoiled mangoes.

SOLUTION

Total Fruit = Base = 2758 kgs of mango

Amount of Spoiled = Percentage = 217 kgs

We want to find Rate% of Spoiled Fruit.

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{217}{2758} = 0.0787 = 7.87\%$$

-:2.18:-

Give the percent of total floor space allocated to each department.

Department	Floor Space	Percent of Floor Space
A	16,000 sq.ft	______
B	20,000 sq. ft	______
C	28,000 sq.ft	______

SOLUTION

First compute the total floor space of the building:

$$16000 + 20000 + 28000 = 64000 \text{ sq ft}$$

Now find the percent of the total that each department occupies:

Department	Floor Space	Percent of Floor
A	16000 sq ft	$\frac{16000}{64000} = 0.25 = 25\%$
B	2000 sq ft	$\frac{20000}{64000} = 0.3125 = 31.25\%$
C	28000 sq ft	$\frac{28000}{64000} = 0.4375 = 43.75\%$

-:2.19:-

Allocate a utility cost of Rs. 4,50,000 for the building just described in question 2.18.

SOLUTION

Here we allocate a utility cost Rs. 450000 for the building to different departments according to their percent.

A $$25\% \text{ of } 450000 = \frac{25}{100} \times 450000 = 0.25 \times 450000$$
$$= \text{Rs.}112500$$

B $$31.25\% \text{ of } 450000 = \frac{31.25}{100} \times 450000 = 0.3125 \times 450000$$
$$= \text{Rs.}140625$$

C $$43.75\% \text{ of } 450000 = \frac{43.75}{100} \times 450000 = 0.4375 \times 450000$$
$$= \text{Rs.}196875$$

-:2.20:-

The family budget is as follows:

Food .... Rs. 10000
Utilities .... Rs. 6000
Education .... Rs. 3500
Clothing .... Rs. 2000
Other expenses .... Rs. 4500

Calculate the percent that each expenditure bears to the total budget.

SOLUTION

First of all calculate the total budget of family.

Food	= Rs. 10000
Utility	= Rs. 6000
Education	= Rs. 3500
Clothing	= Rs. 2000
Other Expenses	= Rs. 4500
Total	**= Rs. 26000**

Now we find percent of each expenditure.

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}}$$

$$\text{Food} = \frac{10000}{26000} = 0.3846 = 38.46\%$$

$$\text{Utility} = \frac{6000}{26000} = 0.2308 = 23.08\%$$

$$\text{Education} = \frac{3500}{26000} = 0.1346 = 13.46\%$$

$$\text{Clothing} = \frac{2000}{26000} = 0.0769 = 7.69\%$$

$$\text{Other Expenses} = \frac{4500}{26000} = 0.1731 = 17.31\%$$

-:2.21:-

Pick quick collected Rs. 5896 in sales tax in March with a tax rate of 5 ½%. What were the total sales for March?

SOLUTION

Here Percentage = Total Tax Collected = Rs. 5896

Rate% = 5 ½% = 5.50%

Base = Total Sales = ?

$$\text{Rate\%} = \frac{\text{Percentage}}{\text{Base}}$$

$$= \frac{5896}{5.5\%} = \frac{5896 \times 100}{5.5} = \frac{5896 \times 1000}{55} = \text{Rs.}107200$$

-:2.22:-

Imtiaz is a real estate sales man and earns 3% of his total sales. What

a) level of sales will give him an income of Rs. 18000 per month.

b) level of sales will Imtiaz need if he wishes monthly income of Rs. 25000?

SOLUTION

(a) Rate% = 3%; Percentage = Income per month = Rs. 18000

$$\text{Level of Sales} = \text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{18000}{3\%} = \frac{18000 \times 100}{3} = \text{Rs. } 600000$$

(b) Rate% = 3%, Percentage = Income per month = Rs. 25000

$$\text{Level of Sales} = \text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{25000}{3\%} = \frac{25000 \times 100}{3} = \text{Rs. } 833333.33$$

-:2.23:-

In analyzing the sizes sold in the men's shoe department, a store found that 1000 pairs of shoes size 7 ½ were sold. This was 8% of the total shoe sales. What was the total number of pairs sold?

SOLUTION

Here Rate% = 8%;

Percentage = No of pairs of shoes of size 7 ½ = 1000 pairs

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{1000}{8\%} = \frac{1000 \times 100}{8} = \frac{100000}{8} = 12500$$

Total No of Pair of Shoes Sold = 12500 pairs of shoes

-:2.24:-

If the sales tax rate is 2% of the sales and the amount of tax paid for the month was Rs. 804.95, calculate the month's sales.

SOLUTION

Here Percentage = The amount of tax paid = Rs. 804.95

Rate% = 2%

Base = Total Sales for the month

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{804.95}{2\%} = \frac{804.95 \times 100}{8} = \frac{80495}{2} = \text{Rs. } 40247.5$$

-:2.25:-

A metal alloy contains 62.8 pounds of zinc. This is 6.5% of the total weight of the alloy. What is the total weight.

SOLUTION

Percentage of Zinc = 62.8 pounds

Rate% = 6.5%

Requirement is to find total weight of Alloy, that is; Base

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{62.8}{6.5\%} = \frac{62.8 \times 100}{6.5} = \frac{62800}{65} = 966.154 \text{ pounds}$$

-:2.26:-

Ali's payroll deduction for one month came to Rs. 5234, whish was 15% of his total salary. Calculate his gross pay.

SOLUTION

Amount of Payroll Deduction = Percentage = Rs. 5234

Rate% of Deduction = 15%

$$\text{Base} = \text{Gross Pay} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{5234}{15\%} = \frac{5234 \times 100}{15} = \frac{523400}{15} = \text{Rs. } 34893.33$$

-:2.27:-

In paying an invoice of Rs. 13345, the customer is entitled to deduct 15% for damages and 18% of the balance for prompt payment. Calculate the cash required to settle the bill.

SOLUTION

Base = Paying Invoice = Rs. 13345

Rate% of Deduction for Damages = 15%

Amount of Deduction = Percentage = Base × Rate%

$$= 13345 \times 15\% = 13345 \times \frac{15}{100}$$

$$= 13345 \times 0.15 = \text{Rs. } 2001.75$$

Balance = Invoice – Deduction

$= \text{Rs. } 13345 - \text{Rs. } 2001.75 = \text{Rs. } 11343.25$

Rate of Deduction for Prompt Payment for Balance = 18%

Amount of Deduction = 18% of 11343.25

$$= 11343.25 \times 0.18\% = \text{Rs.}\,2041.785$$

Hence The amount of Cash to Settle the Bill

$$= \text{Rs. } 11343.25 - \text{Rs. } 2041.785 = \text{Rs. } 9301.465$$

-:2.28:-

A family spend 30% on rent 25% on food and 12% on utilities. The family spend Rs. 3800.35 on utilities. Calculate the gross income of family.

SOLUTION

Family's Expenditure on:

Rent = 30%

Food = 25%

Utilities = 12%

Amount of Expenditure on Utilities = Rs. 3800.35

Which is the Percentage and Rate% of Utilities is 12%

Hence

$$\text{Gross Income of Family} = \text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{3800.35}{12\%} = \frac{804.95 \times 100}{12} = \frac{380035}{12}$$

$$= \text{Rs.}\,31669.583$$

-:2.29:-

Daood bought 250 shares of stock at a total Rs. 4850. After six months, the stock declined in value by 15.125%. What was the stock worth after the decline?

SOLUTION

Here Total Worth before Decline = Rs. 4850

Decline (Decrease) = 15.125%

Amount of Decrease = Percentage = Base × Rate%

$$= 4850 \times 15.125\% = 4850 \times \frac{15.125}{100}$$

$$= 4850 \times 0.15125 = \text{Rs.}\,733.5625$$

Stock Worth after 6 month = Rs. 4850 – Rs. 733.5625

= Rs. 4116.4375

OR

Total Worth = Rs. 4850

Decline = 15.125%

Worth rate% after 6 month = (100 – 15.125)% = 84.875%

Stock Worth after six months

$= 84.875\% \text{ of } 4850 = 0.8750 \times 4850 = \text{Rs. } 4116.4375$

-:2.30:-

Last year, the net income of ABC Corporation decreased by 4.351% from its previous high of Rs. 8,868,472. How much did the company earn last year?

SOLUTION

Net Income of Previous Year than Last Year = Rs. 8,867.8462

Decrease in Last Year = 4.351%

Income of Last Year is as follows:

$= (100 - 4.351)\% \text{ of } 8,86,78,472 \Rightarrow 95.649\% \text{ of } 8,86,78,472$

$= 0.95649 \times 8,86,78,472 = \text{Rs. } 84820071.68$

-:2.31:-

Mr. Kaleem's present salary is Rs. 34000 and he receives a 8% increase. What is his new salary?

SOLUTION

Present Salary of Kaleem = Rs. 34,000

Increase = 8%

New Salary = (100% + 8%) of 34000

$$= (108)\% \text{ of } 34000 = \frac{108}{100} \times 34000 = \text{Rs. } 36720$$

-:2.32:-

Fazal Enterprises reduces the price of its Rs. 24000 printer by 12%. What is the new price?

SOLUTION

Old Price of Printer = Rs. 24,000

Reduced Rate% = 12%

New Price of Printer is as follows:

$= (100\% - 12\%)$ of 24000

$$= 88\% \text{ of } 24000 = \frac{88}{100} \times 24000 = 0.88 \times 24000 = \text{Rs. } 21120$$

-:2.33:-

Style and Smile Electronics reduces the price of its monitors from Rs. 3500 to Rs. 3000. What is the percent of decrease?

SOLUTION

Old Price of Monitors = Rs. 3500

New Price of Monitors = Rs. 3000

Reduction = Percentage = Rs. 500

Base = 3500

$$\text{Reduced Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{500}{3500} = 0.142857 = 14.2857\%$$

-:2.34:-

The population of a town increased from 28600 to 31317. What is the percent of increase?

SOLUTION

Old Population of Town = 28600

Present Population of Town = 31317

Increase = Percentage = 2717

Base = 28600

$$\text{Increased Rate\%} = \frac{\text{Percentage}}{\text{Base}} = \frac{2717}{28600} = 0.095 = 9.5\%$$

-:2.35:-

The retail cost of a microcomputer is Rs. 20000 which is 12% over the dealer's cost. What is the dealer's cost?

SOLUTION

Since

Retail Cost of Micro Computer = Rs. 20,000; is 12% more than the Dealer's Cost. The Question is Rs. 20,000 is 112% of What? That is to find the base.

$$\text{Dealer's Cost} = \frac{\text{Percentage}}{\text{Rate\%}} = \frac{20000}{112\%} = \frac{20000 \times 100}{112} = \text{Rs.}17857.14$$

-:2.36:-

A book sells for Rs. 39.95, which is 20% over cost. What is the cost of the book to the book store?

SOLUTION

Since

The Selling Price of Book = Rs. 29.95 is 20% more than cost price.

Here, we are looking for Rs. 29.95 is 120% of What? That is to find the Base

$$\text{Cost Price of Book} = \text{Base} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{29.95}{120\%} = \frac{29.95 \times 100}{120} = \frac{2995}{120} = \text{Rs.}\,2496$$

-:2.37:-

An electric drill is offered for Rs. 1204, which is 14% of the regular price. What is the regular price?

SOLUTION

This is the problem to find Base:

We have Rs. 1204 is 86% of What?

So

$$\text{Regular Price} = \frac{\text{Percentage}}{\text{Rate\%}}$$

$$= \frac{1204}{86\%} = \frac{1204 \times 100}{86} = \frac{120400}{86} = \text{Rs.}1400$$

-:2.38:-

Tariq, a commission agent, was paid one month as follows:

Rs. 320 on sales at the rate of 3%

Rs. 430 on sales at the rate of 5%

Rs. 236.13 on sales at the rate of 6%

Find the total that Tariq sold that month.

SOLUTION

Amount of Commission (Percentage)	Rate%	Sales (Base)
Rs. 320	3%	Rs. 10666.67
Rs. 430	5%	Rs. 8600.00
Rs. 236.13	6%	Rs. 3935.50
	Total Sales	**Rs. 23202.17**

-:2.39:-

Zafar a wholesale shoe saleman, sold:

150 pairs of shoes at Rs. 599.99 per pair
300 pairs of shoes at Rs. 1000.75 per pair
600 pairs of shoes at Rs. 700.00 per pair

If his commission rate is 4.5%, how much does he receive?

SOLUTION

Pairs of Shoes	Rate per Pair	Amount of Sales
150	Rs. 599.99	Rs. 89998.50
300	Rs. 1000.75	Rs. 300225.00
600	Rs. 700.00	Rs. 420000.00

Total Sales = 89998.50 + 300225 + 420000 = Rs. 810223.50

Rate% of Commission = 4.5%

$$\text{Amount of Commission} = 810223.50 \times 4.5\%$$

$$= \frac{810223.50 \times 4.5}{100} = \frac{810223.50 \times 45}{1000}$$

$$= \text{Rs.}\,36460.06$$

-:2.40:-

Dawar sold Rs. 81000 worth of merchandise last month. He is paid commission on the following basis:

5% on the first Rs. 24000.
7% on the second Rs. 30000
9% on the over Rs. 54000

How much did he earn last month?

SOLUTION

Sales	Rate% of Commission	Amount of Commission
Rs. 24000	5%	24000 x 0.05 = Rs. 1200
Rs. 30000	7%	30000 x 0.07 = Rs. 2100
Rs. 54000	9%	54000 x 0.09 = Rs. 4860
		Total Commission = Rs. 8160

-:2.41:-

The ABC company sells electrical items to wholesaler retailers and industrial purchasers. Each purchaser receives a different discount according to the nature of his business. The retailers get 40%, wholesaler 42% and industrial purchasers 47%. How much did each of the following purchasers pay for item listing at Rs. 2030 each?

a) The retailer bought 12 items

b) The wholesaler bought 30 item.

c) The industrial purchaser bought 120 items

SOLUTION

Since different Rate% are given as follows:

40% discount for Retailer

42% discount for Wholesales

47% discount for Industrial

We calculate the payment of each purchaser in the following table.

Purchaser	Price per unit	No of Units Purchased	Sales Amount	Rate% of Discount	Amount of Discount	Payment
Retailer	Rs. 2030	12	Rs. 24360	40%	Rs. 9744	Rs. 14616
Wholesaler	Rs. 2030	30	Rs. 60900	42%	Rs. 25578	Rs. 35322
Industrial	Rs. 2030	120	Rs.243600	47%	Rs.114492	Rs. 29108

Hence Retailers Pay = Rs. 14616

Wholesaler Pay = Rs. 35322

Industrial Pay = Rs. 129108

-:2.42:-

Find the amount of cash necessary to settle the following transactions within the discount period. A bill was received for Rs. 8120 consisting of Rs. 7000 of merchandise and Rs. 1120 for freight

charges. Upon receiving the merchandise Rs. 500 worth was found to be defective and returned. The discount rate is 2%.

SOLUTION

Here	Total Bill	= Rs. 8120
	Less Freight	= Rs. 1120
		= Rs. 7000
	Less Returned Merchandise	= Rs. 500
		= Rs. 6500
	Less Discount (2% of 6500)	= Rs. 130
		= Rs. 6370
	Plus Freight	= Rs. 1120
	Amount Due	= Rs. 7490

-:2.43:-

A commission saleman is paid as follows:

3% on the first Rs. 1000 of sales

4.5% on the next Rs. 3000 of sales

6.5% on all over Rs. 4000 of sales.

Calculate total remuneration of saleman on sales of Rs. 20,325.

SOLUTION

Total Sales = Rs. 20325
Amount of Commission of Rs. 1000 Sales @ 3% = Rs. 30.00
Amount of Commission of Rs. 3000 Sales @ 4.5% = Rs. 135.00
Amount of Commission of Rs. 16325 Sales @ 6.5% = Rs.1061.125
Total Remuneration of Saleman = Rs. 1226.125

-:2.44:-

A saleman sold merchandise to Rs. 73000 during June. He receives 3% on the fist Rs. 15000; 6% on the next Rs. 15000 and 8% on the remainder. What was his commission for the month?

SOLUTION

Total Sales = Rs. 73000
Amount of Commission for Rs. 15000 Sales @ 3% = Rs. 450
Amount of Commission for Rs. 15000 Sales @ 6% = Rs.900
Amount of Commission for Rs. 43000 Sales @ 8% = Rs. 3440
Total Commission = Rs. 4790

EXERCISE NO. 3

-:3.1:-

If Rs. 10,000 is invested for 6 years at an annual simple interest rate of 16%.

a) How much interest will be earned?

b) What is the amount of the investment at the end of the 6 years.

SOLUTION

Here Principal = Rs. 10000, rate = 16%, Time = 6 years

(a)

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100}$$

$$= \frac{10000 \times 16 \times 6}{100} = \text{Rs. } 9600$$

(b)

$$\text{Amount} = \text{Principal} + \text{Simple Interest} = P + I$$

$$= \text{Rs.}10000 + \text{Rs.}9600 = \text{Rs. } 19600$$

-:3.2:-

Find the simple interest and amount for the following:

i) Rs. 2000 for 3 years at 5%. **ii) Rs. 800 for 7 years at 4%.**

iii) Rs. 4000 for 5 years at 6%.

SOLUTION

(i) Principal = Rs. 2000, Rate = 5%

Time = 3 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{2000 \times 5 \times 3}{100} = \text{Rs. } 300$$

$$\text{Amount} = A = \text{Principal} + \text{Simple Interest}$$

$$= \text{Rs. } 2000 + \text{Rs. } 300 = \text{Rs. } 2300$$

(ii) Principal = Rs. 800, Rate = 4%,

Time = 7 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{800 \times 4 \times 7}{100} = \text{Rs. } 224$$

Amount = Principal + Simple Interest

= Rs. 800 + Rs. 224 = Rs. 1024

(iii) Principal = Rs. 4000, Rate = 6%, Time = 5 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{4000 \times 6 \times 5}{100} = \text{Rs. } 1200$$

Amount = Principal + Simple Interest

= Rs. 4000 + Rs. 1200 = Rs. 5200

-:3.3:-

Mr. Asghar borrowed Rs. 5000 for 5 years at 8% simple interest. How much he will repay?

SOLUTION

Principal = Rs. 5000, Rate = 8%, Time = 5 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{5000 \times 8 \times 5}{100} = \text{Rs. } 2000$$

Mr. Asghar will repay

Amount = Principal + Simple Interest

= Rs. 5000 + Rs. 2000 = Rs. 7000

-:3.4:-

Mr. Bashir Ahmad borrowed Rs. 4500 from Habib Bank at 8½% and lent the whole amount at 10% the same day, what would he gained from this after 4 years.

SOLUTION

Principal = Rs. 4500

$$\text{Rate} = 8\frac{1}{2}\% = \frac{17}{2}\%$$

Time = 4 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{4500 \times 17 \times 4}{100 \times 2} = \text{Rs. } 1530$$

Mr. Bashir lented = Rs. 4500, Rate = 10%, Time= 4 year

$$\text{Simple Interest} = \frac{4500 \times 10 \times 4}{100} = \text{Rs. } 1800$$

He will gain = Rs. 1800 - Rs. 1530 = Rs. 270

-:3.5:-

Khalid Mahmood borrowed Rs. 2500 from Sultan for 3½ years at simple interest at 8% per annum. How much Khalid Mahmood has to pay at the end of the period.

SOLUTION

Principal = Rs. 2500, Rate = 8%

$$\text{Time} = 3\frac{1}{2}\text{ years} = \frac{7}{2}\text{ years} = 4\text{ years}$$

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{2500 \times 8 \times 7}{100 \times 2} = \text{Rs. } 700$$

He will pay = Rs. 2500 + Rs. 700 = Rs. 3200

-:3.6:-

What sum would borrow in Rs. 1250 as simple interest at 3% in 3 years.

SOLUTION

Simple Interest = Rs. 1250, Rate = 3%, Time = 3 years

$$\text{Principal} = P = \frac{I \times 100}{r \times t} = \frac{1250 \times 100}{3 \times 3}$$

$$= \text{Rs. } 13888.89 = \text{Rs. } 13889$$

-:3.7:-

Find the principal, if the amount for 5 years at 5% is Rs. 5550.

SOLUTION

Here Amount = Rs. 5550, Rate = 5%, Time = 5 years

$$\text{Principal} = P = \frac{A \times 100}{100 + (r + t)}$$

$$= \frac{5550 \times 100}{100 + (5 \times 5)} = \frac{555000}{125} = \text{Rs. } 4440$$

-:3.8:-

How much should Mr. Arif borrow from Mr. Hanif so that he may have to repay Rs. 4425 after 3 years if Hanif charges a simple interest rate 5%.

SOLUTION

Here Amount = Rs. 4425, Rate = 5%, Time = 3 years

We have

$$\text{Principal} = P = \frac{A \times 100}{100 + (r + t)}$$

$$= \frac{4425 \times 100}{100 + (5 \times 3)} = \frac{442500}{115} = \text{Rs. } 3847.83 = \text{Rs. } 3848$$

-:3.9:-

How much should Mr. Maqbool borrows from a bank so that he may have to repay Rs. 2800 after 2 years if the bank charges a simple interest rate 4%.

SOLUTION

Here Amount = Rs. 2800, Rate = 4%, Time = 2 years

We have

$$\text{Principal} = P = \frac{A \times 100}{100 + (r + t)}$$

$$= \frac{2800 \times 100}{100 + (4 \times 2)} = \frac{280000}{108} = \text{Rs. } 2592.59 = \text{Rs. } 2593$$

-:3.10:-

Find the sum of money that amount to Rs. 2775 in five years Nine months at 4%.

SOLUTION

Here Amount = Rs. 2775, Rate = 4%,

Time = 5 years and 9 month.

$$5 + \frac{9}{12} \text{ year} = 5\frac{3}{4} \text{ year} = \frac{23}{4} \text{ year}$$

We have

$$\text{Principal} = P = \frac{A \times 100}{100 + (r + t)}$$

$$= \frac{2775 \times 100}{100 + (4 \times \frac{23}{4})} = \frac{2775 \times 100}{100 + 23} = \frac{277500}{123}$$

$$\text{Principal} = P = \text{Rs. } 2256.097 = \text{Rs. } 2256$$

-:3.11:-

At what rate of interest, would Rs. 1800 amount to Rs. 2500 in 2 years.

SOLUTION

Here Principal = P = Rs. 1800 and Amount = A = Rs. 2500

Simple Interest = Amount - Principal

= Rs. 2500 - Rs. 1800 = Rs. 700

Time = 2 years

We have

$$\text{Rate} = r = \frac{I \times 100}{P \times t} = \frac{700 \times 100}{1800 \times 2} = 19.44\%$$

-:3.12:-

Find the rate percent of interest, if the simple interest on Rs. 21540 for 4 years 9 months is Rs. 5000.

SOLUTION

Here Simple Interest = I = Rs. 5000

Principal = P = Rs. 21540

Time = 4 years and 9 months

$$4\frac{9}{12}\text{ years} = 4\frac{3}{4}\text{ years} = \frac{19}{4}\text{ years}$$

We have

$$\text{Rate} = r = \frac{I \times 100}{P \times t}$$

$$= \frac{5000 \times 100 \times 4}{21540 \times 19} = \frac{2000000}{409260} = 4.89\%$$

-:3.13:-

At what rate of interest would a sum of money becomes one and a half in 10 years.

SOLUTION

Suppose Sum = Rs. 100

Simple I becomes one and half itself, so Amount A = Rs. 150

Simple Interest = Rs. 150 - Rs. 100 = Rs. 50

Time = 10 years

Hence

$$\text{Rate} = r = \frac{I \times 100}{P \times t} = \frac{50 \times 100}{100 \times 10} = 5\%$$

-:3.14:-

How long will it take for Rs. 1000 to amount Rs. 1180 at 6% per annum simple interest.

SOLUTION

We know that

Amount = Principal + Simple Interest

$A = P + I$

$I = A - P$

Amount = Rs. 1180, Principal = Rs. 1000

Simple Interest = Rs. 1180 - Rs. 1000 = Rs. 180

Rate = 6%

$$\text{Time} = t = \frac{I \times 100}{P + r} = \frac{180 \times 100}{1000 \times 6} = 3 \text{ years}$$

-:3.15:-

In how many years would a sum lent at 8% be doubled.

SOLUTION

Suppose the sum = Rs. 100, Amount = Rs. 200

Simple Interest = I = Rs. 200 - Rs. 100, Rate = r = 8%

Hence

$$\text{Time} = t = \frac{I \times 100}{P \times r} = \frac{100 \times 100}{100 \times 8} = 12\tfrac{1}{2} \text{ years}$$

-:3.16:-

Find the compound interest on Rs. 8000 for 3 years at 5% per annum.

SOLUTION

Principal = Rs. 8000, Rate = r = 5%, Time = n = 3 years

$$\text{Amount} = P\left(1 + \frac{r}{100}\right)^n$$

$$\text{Amount} = 8000\left(1+\frac{5}{100}\right)^3 = 8000\left(\frac{105}{100}\right)^3$$

$$= 8000\,(1.05)^3 = 8000\,(1.157625) = \text{Rs. } 9261$$

Compound Interest = Rs. 9261 - Rs. 8000 = Rs. 1261

-:3.17:-

Find the amount at compound interest of Rs. 12000 at 6% per annum for 2 years and 6 months. Also find the compound interest.

SOLUTION

Principal = Rs. 12000, Rate = r = 6%

Time = n = 2 years 6 months

The number of six month periods in 2 years 6 month are 5, 6% per annum compounded semi-annually is equivalent to 3% every six months. So,

Here n = 5 six months periods

Rate = 3% compounded semi-annually

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 12000\left(1+\frac{3}{100}\right)^5 = 12000\left(\frac{103}{100}\right)^5$$

$$= 12000\,(1.03)^5 = 12000\,(1.15927074)$$

$$= \text{Rs. } 13911.28889 = \text{Rs. } 13911.29$$

-:3.18:-

Find the compound amounts and compound interest for the given investments.

a) Rs. 5000 at 5% compounded annually for ten years.

b) Rs. 5000 at 5% compounded semi annually for ten years.

c) Rs. 5000 at 5% compounded quarterly for ten years.

d) Rs. 8000 at 8% compounded annually for ten years.

e) Rs. 8000 at 8% compounded semi annually for ten years.

f) Rs. 8000 at 8% compounded quarterly for ten years.

SOLUTION

(a) Principal = P = 5000, Rate = r = 5% Compounded annually
Time = n = 10 years

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n = 5000\left(1+\frac{5}{100}\right)^{10}$$

$$= 5000\left(\frac{105}{100}\right)^{10} = 5000(1.05)^{10}$$

$$= 5000(1.628894627) = \text{Rs. } 8144.47$$

Hence Amount = Rs. 8144.47 and Interest = Rs. 3144.47

(b) Principal = P = 5000, Rate = r = 5% Compounded annually
Time = n = 20 six months periods semi-annually

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 5000\left(1+\frac{25}{100}\right)^{20} = 5000\,(1+0.025)^{20}$$

$$= 5000\,(1.163861644) = \text{Rs. } 8193.08$$

Hence Amount = Rs. 8193.08 and Interest = Rs. 3193.08

(c) Principal = P = 5000, Rate = r = 5% per annum
Five percent compounded quarterly = $^{0.05}/_4 = 0.0125$ per quarter.
Since there are four quarters in a year, the number of interest periods is n = 40 quarters. Hence the compound amount is:

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n = 5000\left(1+\frac{1.25}{100}\right)^{40}$$

$$= 5000\,(1+0.0125)^{40} = 5000\,(1.0125)^{40}$$

$$= 5000(1.643619463) = \text{Rs. } 8218.10$$

Hence Amount = Rs. 8218.10 and Interest = Rs. 3218.10

(d) Principal = P = 8000, Rate = r = 8% per annum
Time = n = 10 years

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$\text{Amount} = 8000\left(1+\frac{8}{100}\right)^{10} = 8000\left(\frac{108}{100}\right)^{10}$$

$$= 8000(1.08)^{10} = 8000\ (2.158924997) = \text{Rs. } 17271.40$$

Hence Amount = Rs. 17271.40 and Interest = Rs. 9271.40

(e) Principal = P = 8000, Rate = r = 8% per annum
8% compounded semi-annually is $^8/_2\%$ = 4% each six month.
There are 20 six month periods i.e. n = 20

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^{n}$$

$$= 8000\left(1+\frac{4}{100}\right)^{20} = 8000\left(\frac{104}{100}\right)^{20}$$

$$= 8000(1.04)^{20} = 8000\ (2.191123143)$$

$$= \text{Rs. } 17528.99$$

Hence Amount = Rs. 17528.99 and Interest = Rs. 9528.99

(f) Principal = P = 8000, Rate = r = 8% per annum
8% compounded quarterly is $^{8\%}/_4$ = 2% per quarter.
There are 40 quarters in 10 years. So n = 40 quarters.

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^{n}$$

$$= 8000\left(1+\frac{2}{100}\right)^{40} = 8000\left(\frac{102}{100}\right)^{40}$$

$$= 8000(1.02)^{40} = 8000\ (2.208039664)$$

$$= \text{Rs. } 17664.32$$

Hence Amount = Rs. 17664.32 and Interest = Rs. 9664.32

-:3.19:-

A man deposits Rs. 5000 at the time of his son's birth, in an investment that pays 4% compounded semi annually. How much will be on deposit on the son's 21st birthday.

SOLUTION

Principal = P = 5000, Rate = r = 4%
r = 2% Compounded semi-annually

Time = n = 21 years = 42 six month periods

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^{n} = 5000\left(1+\frac{2}{100}\right)^{42}$$

$$= 5000\,(1.02)^{42}$$

Let $x = (1.02)^{42}$

taking log of both sides

$\log x = 42 \log(1.02)$

$\log x = 42\,(0.0086001717)$

$\log x = 0.361207214$

taking antilog of both sides

$x = 2.29724466$

Hence Amount = 5000 (2.29724466) = Rs. 11448.62

-:3.20:-

Mr. Aslam has invested Rs. 25000 at 6% compounded annually. What amount would be received after 4 years.

SOLUTION

Principal = P = 25000, Rate = r = 6%, Time = n = 4 years

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^{n}$$

$$= 25000\left(1+\frac{6}{100}\right)^{4} = 25000\left(\frac{106}{100}\right)^{2}$$

$$\text{Amount} = 25000\,(1.06)^{4} = 25000\,(1.26247696)$$

$$= \text{Rs. } 31561.92$$

-:3.21:-

On a saving bank account Bank-A pays 3% interest compounded annually, while Bank-B pays 6% interest compounded semi annually on a deposit of Rs. 2000, how much more interest will be earned in 3 years at Bank-B as compared to Bank-A.

SOLUTION

Principal = P = 2000

Bank A:-

Principal = P = Rs. 2000, Rate = 3%, Time = 3 years

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 2000\left(1+\frac{6}{100}\right)^3 = 2000\left(\frac{106}{100}\right)^3$$

$$= 2000\,(1.06)^3 = 2000\,(1.092727) = \text{Rs. } 2185.45$$

Bank B:-

Principal = P = Rs. 2000

The number of six months periods in 3 years are 6 and 6% per annum compounded semi-annually is equal to 3% every six months. So,

Rate = 3% Compounded semi-annually, Time = n = 6

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 2000\left(1+\frac{3}{100}\right)^6 = 2000\left(\frac{103}{100}\right)^6$$

$$= 2000\,(1.03)^6 = 2000\,(1.194052297) = \text{Rs. } 2388.10$$

Bank B will earn more than Bank A

= Rs. 2388.10 - Rs. 2185.45 = Rs. 202.65

-:3.22:-

Find the principal of Rs. 9628.20 due at the end of 8 years if money is worth 6% compounded semi annually.

SOLUTION

We have to find principal

Time = n = 8 x 2 = 16 six month periods

Rate = r = 6% per annum = 3% Semi annually

Amount = Rs. 9628.20

$$P = \frac{A}{\left(1+\frac{r}{100}\right)^n} = \frac{9628.20}{\left(1+\frac{3}{100}\right)^{16}} = \frac{9628.20}{(1+0.03)^{16}} = \frac{9628.20}{(1.03)^{16}}$$

Let $x = (1.03)^{16}$

Taking log of both sides

$\log x = 16 \log (1.03)$

$\log x = 16 (0.012837224)$

$\log x = 0.205395584$

Taking antilog of both sides

$x = 1.604706397$

Hence $$P = \frac{9628.20}{1.604706397} = \text{Rs. } 6000$$

-:3.23:-

The compound interest for 5 years at 5% is Rs. 1000. Find the principal.

SOLUTION

Compound Interest = Rs. 1000, Rate = 5%, Time = 5 years

We have to find principal, Suppose

Principal = P = Rs. 100

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

$$= 100\left(1+\frac{5}{100}\right)^5 = 100\left(\frac{105}{100}\right)^5$$

$$= 100\,(1.05)^5 = 100\,(1.276281563)$$

$$= \text{Rs. } 127.7281563 = \text{Rs. } 127.63$$

Compound Interest = Amount - Principal

= Rs. 127.63 - Rs. 100 = Rs. 27.63

If C-I is Rs. 27.63, Principal = Rs. 100

If C-I is Rs. 1000, then

$$P = \frac{100 \times 1000}{27.63} = \text{Rs. } 3619.25$$

-:3.24:-

In how many years a sum of Rs. 1000 would amount Rs. 1350 at 4% compounded interest.

SOLUTION

Principal = Rs. 1000, Amount = Rs. 1350, Rate = 4%

We have to find n

$$n = \frac{\log A - \log P}{\log\left(1 + \frac{r}{100}\right)}$$

$$= \frac{\log 1350 - \log 1000}{\log\left(1 + \frac{4}{100}\right)} = \frac{\log 1350 - \log 1000}{\log(1.04)}$$

$$= \frac{3.13033 - 3.00000}{0.01703} = \frac{0.13033}{0.01703} = 7.65\text{years}$$

-:3.25:-

In how many years a sum of Rs. 5560 would amount Rs. 7000 at 8% interest compounded semi annually.

SOLUTION

Principal = P = Rs. 5560, Amount = A = Rs. 7000

Rate = 8% per annum = 4% semi-annually

We have to find n

$$n = \frac{\log A - \log P}{\log\left(1 + \frac{r}{100}\right)}$$

$$= \frac{\log 7000 - \log 5560}{\log\left(1 + \frac{4}{100}\right)} = \frac{\log 7000 - \log 5560}{\log(1.04)}$$

$$= \frac{3.84509 - 3.74507}{0.01703} = \frac{0.10002}{0.01703}$$

n = 5.87 = 6 periods of six months = 3 years.

-:3.26:-

In how many years a sum of Rs. 3000 would amount Rs. 6130.43 at 6% compounded quarterly.

SOLUTION

Principal = P = Rs. 3000, Amount = A = Rs. 6130.43

Rate = 6% per annum

6% compounded quarterly = $^{6\%}/_{4} = 1.5\%$

We have to find n

$$n = \frac{\log A - \log P}{\log\left(1 + \frac{r}{100}\right)} = \frac{\log 6130.43 - \log 3000}{\log\left(1 + \frac{1.5}{100}\right)}$$

$$= \frac{\log 6130.43 - \log 3000}{\log(1.015)} = \frac{3.78749038 - 3.477121255}{0.006466042}$$

$$= \frac{0.310369683}{0.006444042} = 47.9999485 = 48 \text{ quarters} = 12 \text{ years}$$

-:3.27:-

At what rate of interest compounded per annum for 5 years will Rs. 2000 amount to Rs. 2250.

SOLUTION

Here Principal = P = Rs. 2000

Amount = A = Rs. 2550

n = 5 years.

We have to find the value of r.

$$\frac{r}{100} = \text{Anti}\log\left[\frac{1}{n}(\log A - \log P\right] - 1 \Rightarrow \frac{r}{100} = \text{Anti}\log\left[\frac{\log A - \log P}{n}\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log\left[\frac{\log 2550 - \log 2000}{5}\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log\left[\frac{3.40654 - 3.30103}{5}\right] - 1 \Rightarrow \frac{r}{100} = \text{Anti}\log\left[\frac{0.10551}{5}\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log(0.021102) - 1 = 1.049788957 - 1$$

$$r = 4.9788957\% = 5\%$$

-:3.28:-

At what rate of interest compounded quarterly for 2½ years will Rs. 2500 amount to Rs. 3900

SOLUTION

Principal = P = Rs. 2500, Amount = A = Rs. 3900

n = 2½ years = 10 periods of three months as interest compounded is quarterly

We have to find the value of r.

$$\frac{r}{100} = \text{Anti}\log\left[\frac{1}{n}(\log A - \log P\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log\left[\frac{\log 3900 - \log 2500}{10}\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log\left[\frac{3.59106 - 3.39794}{10}\right] - 1 \Rightarrow \frac{r}{100} = \text{Anti}\log\left[\frac{0.19312}{10}\right] - 1$$

$$\frac{r}{100} = \text{Anti}\log(0.019312) - 1 = 1.04547 - 1$$

$r = 4.547\%$ quarterly $= 18.18\%$ per annum

-:3.29:-

What is the difference between the compound and simple interest of Rs. 15625 for 4 years at 4%.

SOLUTION

Principal = P = Rs. 15625, Rate = r = 4%, Time = n = 4 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{15625 \times 4 \times 4}{100}$$

Simple Interest = Rs. 2500

Compound Interest

$$\text{Amount} = P\left(1 + \frac{r}{100}\right)^n = 15625\left(1 + \frac{4}{100}\right)^4 = 15625\left(\frac{104}{100}\right)^4$$

$$= 15625\,(1.04)^4 = 15625(1.16985856) = \text{Rs. } 18279.04$$

Compound Interest = Amount - Principal

= Rs. 18279.04 - Rs. 15625 = Rs. 2654.04

Difference between simple and compound interest

= Rs. 2654.04 - Rs. 2500 = Rs. 154.04

-:3.30:-

If the difference between the simple and compound interest for 3 years at 5% is Rs. 61. Find the principal.

SOLUTION

Let the Principal = P = Rs. 100, Rate = r = 5%, Time = n = 3 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{100 \times 5 \times 3}{100} = \text{Rs. } 15$$

Compound Interest

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n = 100\left(1+\frac{5}{100}\right)^3$$

$$= 100\,(1.05)^3 = 100\,(1.157625) = \text{Rs. } 115.76$$

Compound interest = Rs. 115.76 - Rs. 100 = Rs. 15.76

Difference between simple and compound interest

= Rs. 15.76 - Rs. 15 = Rs. 0.76

If difference is Rs. 0.76, then principal = Rs. 100

If difference is Rs. 61, then

$$\text{Principal} = \frac{100 \times 61}{0.76} = \text{Rs. } 8026.32$$

-:3.31:-

The difference between the simple and compound interest on a certain sum is Rs. 2.50 for 2 years at 5%. Find the sum.

SOLUTION

Let the Principal = P = Rs. 100, Rate = r = 5%, Time = n = 2 years

$$\text{Simple Interest} = I = \frac{P \times r \times t}{100} = \frac{100 \times 5 \times 2}{100} = \text{Rs. } 10$$

Compound Interest

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n = 100\left(1+\frac{5}{100}\right)^2 = 100\left(\frac{105}{100}\right)^2$$

$$= 100\,(1.05)^2 = 100\,(1.1025) = \text{Rs. } 110.25$$

Compound interest = Rs. 110.25 - Rs. 100 = Rs. 10.25

Difference between simple and compound interest

= Rs. 10.25 - Rs. 10 = Rs. 0.25

If difference is Rs. 0.25, then principal = Rs. 100

If difference is Rs. 2.5, then

$$\text{Principal} = \frac{100 \times 2.50}{0.25} = \text{Rs. } 1000$$

EXERCISE NO. 4

-:4.1:-

Rs. 500 a year for ten years at 4% compounded annually.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 500, i = 4% and n = 10

$$S = 500\left[\frac{(1+0.04)^{10} - 1}{0.04}\right]$$

$$= 500\left[\frac{1.48024428 - 1}{0.04}\right] = 500\left[\frac{0.48024428}{0.04}\right]$$

$$= 500(12.006107) = \text{Rs. } 6003.05$$

-:4.2:-

Rs. 800 a quarter for 5 years at 4% compounded quarterly.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 800, i = $^{4\%}/_4$ = 1% and n = 20 periods

$$S = 800\left[\frac{(1+0.01)^{20} - 1}{0.01}\right]$$

$$= 800\left[\frac{(1.01)^{20} - 1}{0.01}\right] = 800\left[\frac{1.22019004 - 1}{0.01}\right]$$

$$= 800\left[\frac{0.22019004}{0.01}\right] = 800(22.019004)$$

$$= \text{Rs. } 17615.20$$

-:4.3:-

Rs. 100 a month for 5 years at 6% compounded monthly.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 100, $i = {}^{6\%}/_{12} = {}^{1\%}/_{2} = 0.005$ per periods and $n = 60$

$$S = 100\left[\frac{(1+0.005)^{60} - 1}{0.005}\right]$$

$$= 100\left[\frac{(1.005)^{60} - 1}{0.005}\right] = 100\left[\frac{1.34885015 - 1}{0.005}\right]$$

$$= 100\left[\frac{0.34885015}{0.005}\right] = 100(69.77003) = \text{Rs. } 6977.00$$

-:4.4:-

A man invest Rs. 300 at the end of each month in a fund which pays 4% compounded semiannually. How much does he have just after the tenth deposit.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 300, $i = {}^{4\%}/_{2} = 2\%$ per periods and $n = 10$

$$S = 300\left[\frac{(1+0.02)^{10} - 1}{0.02}\right]$$

$$= 300\left[\frac{(1.02)^{10} - 1}{0.02}\right] = 300\left[\frac{1.21899442 - 1}{0.02}\right]$$

$$= 300\left[\frac{0.21899442}{0.02}\right] = 300(10.949721)$$

$$= \text{Rs. } 3284.92$$

-:4.5:-

Find the amount of Rs. 250 invested at the end of each of 5 successive years at 6% interest compounded annually.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 250, i = 6% and n = 5

$$S = 250\left[\frac{(1+0.06)^5 - 1}{0.06}\right]$$

$$= 250\left[\frac{(1.06)^5 - 1}{0.06}\right] = 250\left[\frac{1.3382258 - 1}{0.06}\right]$$

$$S = 250\left[\frac{0.3382258}{0.06}\right] = 250(5.637096667) = \text{Rs. } 1409.27$$

-:4.6:-

Miss Nasima saved Rs. 540 per year which she deposited in a saving bank at the end of each year. If the bank paid 2% interest compounded annually on all deposits, what was the amount on deposit at the end of 11 years?

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 540, i = 2% and n = 11

$$S = 540\left[\frac{(1+0.02)^{11} - 1}{0.02}\right]$$

$$= 540\left[\frac{(1.02)^{11} - 1}{0.02}\right] = 540\left[\frac{1.24337431 - 1}{0.02}\right]$$

$$= 540\left[\frac{0.24337431}{0.02}\right] = 540(12.1687155) = \text{Rs. } 6571.11$$

-:4.7:-

Mr. Khalid wishes to save money to take a trip. If he deposits Rs. 150 at the end of each month for 24 months in an investment that pays 12% compounded monthly, how much will he have on deposit.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 150, $i = {}^{12\%}/_{12} = 1\%$ per periods and n = 24

$$S = 150\left[\frac{(1+0.01)^{24} - 1}{0.01}\right]$$

$$= 150\left[\frac{(1.01)^{24} - 1}{0.01}\right] = 150\left[\frac{1.26973465 - 1}{0.01}\right]$$

$$= 150\left[\frac{0.26973465}{0.01}\right] = 150(26.973465) = \text{Rs. } 4046.02$$

-:4.8:-

Find the amount of an ordinary annuity of Rs. 4400 per quarter for 5 years at the rate of 16% per annum compounded quarterly.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 4400, $i = {}^{16\%}/_{4} = 4\%$ per periods and n = 20

$$S = 4400\left[\frac{(1+0.04)^{20} - 1}{0.04}\right]$$

$$= 4400\left[\frac{(1.04)^{20} - 1}{0.04}\right] = 4400\left[\frac{2.19112314 - 1}{0.04}\right]$$

$$= 4400\left[\frac{1.19112314}{0.04}\right] = 4400(29.7780785)$$

$$= \text{Rs. } 131023.55$$

-:4.9:-

Mrs. Khurshid deposit Rs. 5555 at the end of each six months for 4 years and 6 months at 10% compounded semiannually. Find the amount of the investment.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 5555, i = $^{10\%}/_2$ = 5% per periods and n = 9

$$S = 5555\left[\frac{(1+0.05)^9 - 1}{0.05}\right]$$

$$= 5555\left[\frac{(1.05)^9 - 1}{0.05}\right] = 5555\left[\frac{1.55132822 - 1}{0.05}\right]$$

$$= 5555\left[\frac{0.55132822}{0.05}\right] = 5555(11.0265644) = \text{Rs. } 61252.565$$

-:4.10:-

Find the accumulated value of Rs. 5000 invested at the end of each quarter year for 5 years at 8% compounded quarterly.

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Here R = Rs. 5000, i = 2% and n = 20 quarters

$$S = 5000\left[\frac{(1+0.02)^{20} - 1}{0.02}\right]$$

$$= 5000\left[\frac{(1.05)^{20} - 1}{0.02}\right] = 5000\left[\frac{1.485947396 - 1}{0.02}\right]$$

$$= 5000\left[\frac{0.485947396}{0.02}\right] = 5000(24.2973698) = \text{Rs. } 121486.85$$

-:4.11:-

A man sets a side Rs. 200 at the beginning of each year towards a fund for his son's college education. If the money is invested at 4% per year, how much will he accumulated at the end of 10 years?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 200, i = 4% and n = 10

$$S = 200\left[\frac{(1+0.04)^{10+1}-1}{0.04}\right] - 200$$

$$= 200\left[\frac{(1+0.04)^{10+1}-1}{0.04}\right] - 200$$

$$= 200\left[\frac{(1.04)^{11}-1}{0.04}\right] - 200 \Rightarrow 200\left[\frac{1.53945406-1}{0.04}\right] - 200$$

$$= 200\left[\frac{0.53945406}{0.04}\right] - 200 = 200(13.4863515) - 200$$

= Rs. 2697.27 – Rs. 200 = Rs. 2497.27

-:4.12:-

If a payment of Rs. 500 is made today and a like payment each year for 5 years, how much will be on deposit at the end of 4 years, if the interest rate is 5% compounded annually?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 500, i = 5% and n = 5

$$S = 500\left[\frac{(1+0.05)^{5+1}-1}{0.05}\right] - 500$$

$$S = 500\left[\frac{(1.05)^{6}-1}{0.05}\right] - 500 = 500\left[\frac{1.34009564-1}{0.05}\right] - 500$$

$$= 500\left[\frac{0.34009564}{0.05}\right] - 500 \Rightarrow 500(6.8109128) - 500$$

= Rs. 3400.96 – Rs. 500 = Rs. 2900.96

-:4.13:-

How much money would be accumulated at the end of 23 years by a man if he invested Rs. 325 on the first day of each year at 4% interest compounded annually?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 325, i = 4% and n = 23

$$S = 325\left[\frac{(1+0.04)^{23+1}-1}{0.04}\right] - 325 \Rightarrow 325\left[\frac{(1.04)^{24}-1}{0.04}\right] - 325$$

$$= 325\left[\frac{2.56330416-1}{0.04}\right] - 325 \Rightarrow 325\left[\frac{1.56330416}{0.04}\right] - 325$$

$$= 325(39.082604) - 325 = \text{Rs. } 12701.85 - \text{Rs. } 325 = \text{Rs. } 12376.85$$

-:4.14:-

On his fifty first birthday and each third month thereafter, Dr. Habib Ullah invested Rs. 475 at 4% interest compounded quarterly. At the end of his fifty seven year, what was the amount on deposit?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 475, i = $^{4\%}/_{4}$=1% per period and n = 24

$$S = 475\left[\frac{(1+0.01)^{24+1}-1}{0.01}\right] - 475$$

$$= 475\left[\frac{(1.01)^{25}-1}{0.01}\right] - 475 = 475\left[\frac{1.28243200-1}{0.01}\right] - 475$$

$$= 475\left[\frac{0.28243200}{0.01}\right] - 475 = 475(28.2432) - 475$$

$$S = \text{Rs. } 13415.52 - \text{Rs. } 475 = \text{Rs. } 12940.52$$

-:4.15:-

Mr. Aslam bought a SONY T.V. by paying Rs. 350 each month for 12 months, beginning from now. If money is worth 12% compounded monthly, what was the selling price of the T.V. on each payment?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 350, $i = {}^{12\%}/_{12} = 1\%$ per period and n = 12

$$S = 350\left[\frac{(1+0.01)^{12+1}-1}{0.01}\right] - 350 \Rightarrow 350\left[\frac{(1.01)^{13}-1}{0.01}\right] - 350$$

$$= 350\left[\frac{1.13809328-1}{0.01}\right] - 350 \Rightarrow 350\left[\frac{0.13809328}{0.01}\right] - 350$$

$$= 350(13.809328) - 350 = \text{Rs. } 4833.26 - \text{Rs. } 350 = \text{Rs. } 4483.26$$

-:4.16:-

Find the amount of an annuity due of Rs. 200 paid at the beginning of each 6 month period for 8 years if the interest rate is 6% compounded semiannually.

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Here R = Rs. 200, $i = {}^{6\%}/_{2} = 3\%$ per period and n = 16

$$S = 200\left[\frac{(1+0.03)^{16+1}-1}{0.03}\right] - 200$$

$$= 200\left[\frac{(1.03)^{17}-1}{0.03}\right] - 200 = 200\left[\frac{1.65284763-1}{0.03}\right] - 200$$

$$= 200\left[\frac{0.65284763}{0.03}\right] - 200 = 200(21.76158767) - 200$$

$$= \text{Rs. } 4352.32 - \text{Rs. } 200 = \text{Rs. } 4152.32$$

-:4.17:-

A house is rented for Rs. 900 per month, with each month's rent payable in advance. If the interest rate is 12% compounded monthly and the rent is deposited in an account, what is the amount of rent for one year?

SOLUTION

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right]-R$$

Here R = Rs. 900, $i = {}^{12\%}/_{12} = 1\%$ per period and n = 12

$$S = 900\left[\frac{(1+0.01)^{12+1}-1}{0.01}\right]-900 \Rightarrow 900\left[\frac{(1.01)^{13}-1}{0.01}\right]-900$$

$$=900\left[\frac{1.13809328-1}{0.01}\right]-900 \Rightarrow 900\left[\frac{0.13809328}{0.01}\right]-900$$

$$=900(13.809328)-900 = \text{Rs. } 12428.40 - \text{Rs. } 900 = \text{Rs. } 11528.40$$

-:4.18:-

Find the present value of an annuity of Rs. 100 paid at the end of each year for 17 years if the interest rate is 7%, compounded annually.

SOLUTION

By Formula:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Here R = Rs. 100, i = 7% and n = 17

$$P = 100\left[\frac{1-(1+0.07)^{-17}}{0.07}\right] = 100\left[\frac{1-(1.07)^{-17}}{0.07}\right]$$

$$(1.07)^{-17} = 0.31657439$$

So $P = 100\left[\frac{1-0.31657439}{0.07}\right]$

$$=100\left[\frac{0.68342561}{0.07}\right] = 100(9.763223) = \text{Rs. } 976.32$$

By Table: From table:

$$P = R.^{a}\overline{n}|i$$

$$^{a}\overline{17}|7\% = 9.7632299$$

$$P = 100(9.763223) = Rs.\ 976.32$$

:4.19:-

Find the present value of an annuity of Rs. 100 paid at the end of each year for 8 years if the interest rate is 8% compounded semiannually.

SOLUTION

By Formula;

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Here R = Rs. 100, $i = {}^{8\%}/_{2} = 4\%$ per period and n = 16

$$P = 100\left[\frac{1-(1+0.04)^{-16}}{0.04}\right] = 100\left[\frac{1-(1.04)^{-16}}{0.04}\right]$$

$$(1.04)^{-16} = 0.53390818$$

So $$P = 100\left[\frac{1-0.53390818}{0.04}\right]$$

$$= 100\left[\frac{0.46609182}{0.04}\right] = 100(11.6522955)$$

$$= Rs.\ 1165.23$$

By Table: From table:

$$^{a}\overline{16}|4\% = 11.65229561$$

$$P = R.^{a}\overline{n}|4\%$$

$$= 100(11.6522955) = Rs.\ 1165.23$$

:4.20:-

A man wishes to deposit enough money today so that his son can withdraw Rs. 2500 at the end of each year for ten years. How much he deposit, if the interest rate is 6% compounded annually.

SOLUTION

By Formula:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Here R = Rs. 2500, i = 6% and n = 10

$$P = 2500\left[\frac{1-(1+0.06)^{-10}}{0.06}\right] = 2500\left[\frac{1-(1.06)^{-10}}{0.06}\right]$$

$$(1.06)^{-10} = 0.55839478$$

So $$P = 2500\left[\frac{1-0.55839478}{0.06}\right] = 2500\left[\frac{0.44160522}{0.06}\right]$$

$$= 2500(7.360087) = \text{Rs. } 18400.22$$

By Table: From table:

$$a_{\overline{10}|6\%} = 7.360087$$

$$P = R \cdot a_{\overline{10}|6\%}$$

$$= 2500(7.360087) = \text{Rs. } 18400.22$$

:4.21:-

What is the present value of an annuity if the size of each payment is Rs. 2000 payable at the end of each quarter for eight years at an interest rate of 7% compounded quarterly.

SOLUTION

Here R = Rs. 2000, i = $^{7\%}/_4$ = 1.75%, n = 32 quarters

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2000\left[\frac{1-(1+0.0175)^{-32}}{0.0175}\right] = 2000\left[\frac{1-(1.0175)^{-32}}{0.0175}\right]$$

Let $x = (1.0175)^{-32}$

Taking log of both sides

$$\log x = -32\log(1.0175)$$

$$= -32(0.007534417) = -0.241101372$$

$$= -1 + 1 - 0.241101372 = \bar{1}.758898628$$

Taking antilog of both sides

$$x = 0.5739824685$$

$$P = 2000\left[\frac{1-0.5739824685}{0.0175}\right] = 2000\left[\frac{0.426017532}{0.0175}\right]$$
$$= 2000[24.34385897] = \text{Rs. } 48687.72$$

:4.22:-

What is the present value of an annuity of Rs. 300 per year if the first payment is received today and yearly for a total of 10 consecutive payments, and money is worth 5% interest compounded annually?

SOLUTION

By Formula:

$$P = R\left[1 + \frac{1-(1+i)^{-(n-1)}}{i}\right]$$

Here R = Rs. 300, i = 6% and n = 10

$$P = 300\left[1 + \frac{1-(1+0.06)^{-(10-1)}}{0.06}\right] = 300\left[1 + \frac{1-(1.06)^{-9}}{0.06}\right]$$
$$= 300\left[1 + \frac{1-0.591189846}{0.06}\right] = 300\left[1 + \frac{0.40810154}{0.06}\right]$$
$$= 300(1+6.801692333) = 300(7.801692333) = \text{Rs. } 2340.51$$

By Table: From table:

$$P = R\left[1 + a_{\overline{n-1}|}i\%\right] = 300\left[1 + a_{\overline{9}|}6\%\right]$$

$$a_{\overline{9}|}6\% = 6.80169227$$

$$P = 300(1+6.80169227) = 300(7.80169227) = \text{Rs. } 2340.51$$

-:4.23:-

The amount of an annuity for 12 years is Rs. 80000. What would be the size of the quarterly installment if the interest rate is 12% compounded quarterly?

SOLUTION

By Formula:

$$R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]}$$

We have S = Rs. 80000. $i = {}^{12\%}/_4 = 3\%$, n = 48 periods

$$R = \frac{80000}{\left[\frac{(1+0.03)^{48} - 1}{0.03}\right]} = \frac{80000}{\left[\frac{(1.03)^{48} - 1}{0.03}\right]} = \frac{80000 \times 0.03}{4.13225188 - 1}$$

$$= \frac{80000 \times 0.03}{3.13225188} = \frac{2400}{3.13225188} = \text{Rs. } 766.22$$

By Table:

$$R = \frac{S}{s_{\overline{n}|i}} = \frac{80000}{s_{\overline{48}|3\%}}$$

$$s_{\overline{48}|3\%} = 104.40839598$$

$$R = \frac{80000}{104.40839598} = \text{Rs. } 766.22$$

-:4.24:-

Mr. Amjad wants to accumulate Rs. 60000 in 8 years. He makes equal deposits at the end of each 6 months in an account. The rate of interest is 8% compounded semiannually. Find the value of each deposit.

SOLUTION

By Formula:

$$R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]}$$

We have S = Rs. 60000, $i = {}^{8\%}/_2 = 4\%$ per period, n = 16 periods

$$R = \frac{60000}{\left[\frac{(1+0.04)^{16} - 1}{0.04}\right]} = \frac{60000 \times 0.04}{(1.04)^{16} - 1}$$

$$= \frac{2400}{1.87298125 - 1} = \frac{2400}{0.87291825} = \text{Rs. } 2749.40$$

By Table:

$$R = \frac{S}{s_{\overline{n}|i}} = \frac{60000}{s_{\overline{16}|4\%}}$$

$$s_{\overline{16}|4\%} = 21.82453114$$

$$R = \frac{60000}{21.82453114} = \text{Rs. } 2749.40$$

-:4.25:-

Mr. Latif borrowed Rs. 200,000 from his friend to purchase a car. He promised to pay back the loan in semiannual equal installments in 3 years with interest rate at 8% compounded semiannually. If first payment is start at the end of first six monthly period, what would be the amount of each installment?

SOLUTION

By Formula: $$R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

We have P = Rs. 200000, $i = {}^{8\%}/_2 = 4\%$ per period, n = 6 periods

$$R = \frac{200000}{\left[\frac{1-(1+0.04)^{-6}}{0.04}\right]} = \frac{200000 \times 0.04}{1-(1.04)^{6}}$$

$$= \frac{8000}{1-0.79031453} = \frac{8000}{0.20968547} = \text{Rs. } 38152.38$$

By Table:

$$R = \frac{P}{a_{\overline{n}|i}} = \frac{200000}{a_{\overline{6}|4\%}}$$

$$a_{\overline{6}|4\%} = 5.24213686$$

$$R = \frac{200000}{5.24213686} = \text{Rs. } 38152.38$$

-:4.26:-

How many semi annual payments of Rs. 200 each to in a bank in the form of an ordinary annuity will accumulate Rs. 6000 if the interest rate is 8%.

SOLUTION

By Formula:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

We have S = Rs. 6000, R = Rs. 200, i = 8%

$$6000 = 200\left[\frac{(1+0.08)^n - 1}{0.08}\right]$$

$$6000 = 200\left[\frac{(1.08)^n - 1}{0.08}\right]$$

$$(1.08)^n - 1 = \frac{6000 \times 0.08}{200}$$

$$(1.08)^n = 2.4 + 1 = 3.4$$

Taking log of both sides

$$n \log(1.08) = \log 3.4$$

$$n = \frac{\log 3.4}{\log 1.08} = \frac{0.531478917}{0.33423755} = 15.9 = 16$$

By Table:

$$S = R.s_{\overline{n}|i}$$

$$s_{\overline{n}|i} = \frac{S}{R} = \frac{6000}{200} = 30$$

We consults the table under 8% and we find the value 30.324283 which gives n = 16.

-:4.27:-

The cash price of a car is Rs. 150000. If the interest rate is 24% compounded quarterly. How many quarterly payments he will take to pay off car, when quarterly payments of Rs. 15000 are made.

SOLUTION

By Formula:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

We have P = Rs. 150000, R = Rs. 15000, i = $^{24\%}/_4$ = 6%

$$150000 = 15000\left[\frac{1-(1+0.06)^{-n}}{0.06}\right]$$

$$= 1-(1.06)^{-n} = \frac{150000 \times 0.06}{15000}$$

$$1-(1.06)^{-n} = 0.6$$

$$(1.06)^{-n} = 1 - 0.6 = 0.4$$

Taking log of both sides

$$-n \log(1.06) = \log 0.4$$

$$-n = \frac{\log 0.4}{\log 1.06} = \frac{-0.39794000}{0.025305865}$$

$$-n = 16 \text{ approx.} = 16 \text{ approx.}$$

By Table:

$$P = R.a_{\overline{n}|i}$$

$$150000 = 15000\, a_{\overline{n}|6\%}$$

$$a_{\overline{n}|6\%} = \frac{150000}{15000} = 10$$

In table look down the column 6% for an entry close to 10. The nearest is 10.10589527 which corresponds to n = 16.

-:4.28:-

Mr. Usman wants to deposit his savings of Rs. 48000 in a bank. The bank offers 8% interest compounded semi annually. Find the number of withdrawals, if he draw Rs. 2400 at the end of each six months.

SOLUTION

By Formula:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

We have P = Rs. 48000, R = Rs. 2400, $i = {}^{8\%}/_{2} = 4\%$

$$48000 = 2400\left[\frac{1-(1+0.04)^{-n}}{0.04}\right]$$

$$48000 = 2400\left[\frac{1-(1.04)^{-n}}{0.04}\right]$$

$$1-(1.04)^{-n} = \frac{48000 \times 0.04}{2400}$$

$$1-(1.04)^{-n} = 0.8$$

$$(1.04)^{-n} = 1 - 0.8 = 0.2$$

Taking log of both sides

$$-n\log(1.04) = \log 0.2$$

$$-n = \frac{\log 0.2}{\log 1.04} = \frac{-0.698970004}{0.017033339}$$

$$-n = -41 \Rightarrow n = 41$$

By Table:

$$P = R.a_{\overline{n}|i}$$

$$48000 = 2400\ a_{\overline{n}|4\%}$$

$$a_{\overline{n}|4\%} = \frac{48000}{2400} = 20$$

In table look down the column 4% for an entry close to 20. The nearest is 19.99305181 which corresponds to n = 41.

-:4.29:-

Mr. Zahid invests Rs. 540 at the end of each month amounts to Rs. 12644 in 1½ year. Find the nominal rate of interest.

SOLUTION

If S, R and n are given the rate of interest per period i can not be found with the help of logarithm as the value of the factor $[(1+i)^n - 1]$ cannot be determined if i is unknown. The possible solution will be found by use of annuity tables.

Here R = Rs. 540, S = Rs. 12644, n = 18 months

$$S = R.s_{\overline{n}|i}$$

$$13000 = 540\ s_{\overline{18}|i\%}$$

$$s_{\overline{18}|i} = \frac{12644}{540} = 23.41481481$$

In table -2 look from left to right opposite n = 30, searching for the value nearest 23.41443547. Which occurs under 3%. The nearest rate is 3% per month or 12 × 3% = 36% compounded monthly.

-:4.30:-

An annuity of Rs. 8800 payable at the end of each year amounts to Rs. 120000 in 10 years. Find the interest of the annuity.

SOLUTION

Here S = Rs. 120000, R = Rs. 8800, n = 10

$$S = R.{}^{s}\overline{n}\rceil i$$

$$120000 = 8800\ {}^{s}\overline{10}\rceil i$$

$${}^{s}\overline{10}\rceil i = \frac{120000}{8800} = 13.63636364$$

In table -2 look from left to right opposite n = 10, the nearest value is 13.81644796 which occurs under 7% per year.

-:4.31:-

The GO company is expected to pay Rs. 4.0 every six months on a share of its stock what is the present value of this stock if money is worth 5% compounded semi annually?

SOLUTION

Here R = Rs. 4.0, n = $^{5\%}/_{2}$=2½%

$$P = \frac{R}{i} = \frac{4.0}{0.025} = \frac{4000}{25} = Rs.160$$

-:4.32:-

Find the present value of a company's stock, which is expected to pay Rs. 200 every three month, if money is worth 6% compounded quarterly.

SOLUTION

Here R = Rs. 200, n = $^{6\%}/_{4}$=1½%

$$P = \frac{R}{i} = \frac{200}{0.015} = Rs.13333.33$$

EXERCISE NO. 5

SET – A

-:5.1:-

If $f(x) = 5x + 3$, find $f(1)$, $f(2)$, $f(3)$

SOLUTION

$$f(x) = 5x + 3$$
$$f(1) = 5(1) + 3 = 4 + 3 = 8$$
$$f(2) = 5(2) + 3 = 10 + 3 = 13$$
$$f(3) = 5(3) + 3 = 15 + 3 = 18$$

-:5.2:-

If $f(x) = 2r - 5$, find $f(1)$, $f(10)$, $f(100)$

SOLUTION

$$f(r) = 2r - 5$$
$$f(1) = 2(1) - 5 = 2 - 5 = -3$$
$$f(10) = 2(10) - 5 = 20 - 5 = 15$$
$$f(100) = 2(100) - 5 = 200 - 5 = 195$$

-:5.3:-

If $f(t) = 6t + 4$, find $f(-1/2)$, $f(1/2)$, $f(3/2)$

SOLUTION

$$f(t) = 6t + 4$$
$$f(-\tfrac{1}{2}) = 6(-\tfrac{1}{2}) + 4 = -3 + 4 = 1$$
$$f(\tfrac{1}{2}) = 6(\tfrac{1}{2}) + 4 = 3 + 4 = 7$$
$$f(\tfrac{3}{2}) = 6(\tfrac{3}{2}) + 4 = 9 + 4 = 13$$

-:5.4:-

If $y = 2x + 3$, find y when $x = 1, 2, 3, 4$

SOLUTION

$$y = 2x + 3$$
$$y = 2(1) + 3 = 2 + 3 = 5$$
$$y = 2(2) + 3 = 4 + 3 = 7$$
$$y = 2(3) + 3 = 6 + 3 = 9$$
$$y = 2(4) + 3 = 8 + 3 = 11$$

-:5.5:-

If y = -3x + 5, find y when x = -1, -2, -3, -4

SOLUTION

$$y = -3x + 5$$

$$y = -3(-1) + 5 = 3 + 5 = 8$$

$$y = -3(-2) + 5 = 6 + 5 = 11$$

$$y = -3(-3) + 5 = 9 + 5 = 14$$

$$y = -3(-4) + 5 = 12 + 5 = 17$$

-:5.6:-

For the line y = 4x + 2, compute the following table.

x	0	1	2	3	4	5	6	7	8	9	10
y											

SOLUTION

$$y = 4x + 2$$

$y = 4(0) + 2 = 0 + 2 = 2$, $y = 4(1) + 2 = 4 + 2 = 6$
$y = 4(2) + 2 = 8 + 2 = 10$, $y = 4(3) + 2 = 12 + 2 = 14$
$y = 4(4) + 2 = 16 + 2 = 18$, $y = 4(5) + 2 = 20 + 2 = 22$
$y = 4(6) + 2 = 24 + 2 = 26$, $y = 4(7) + 2 = 28 + 2 = 30$
$y = 4(8) + 2 = 32 + 2 = 34$, $y = 4(9) + 2 = 36 + 2 = 38$
$y = 4(10) + 2 = 40 + 2 = 42$

X	0	1	2	3	4	5	6	7	8	9	10
Y	2	6	10	14	18	22	26	30	34	38	42

-:5.7:-

Find the equation to the straight line passing through (0, 0) and (2, -2)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 0$, $y_1 = 0$, $x_2 = 2$, $y_2 = -2$

$$(y - 0) = \frac{-2 - 0}{2 - 0}(x - 0)$$

$$y = -1(x - 0)$$

$$y = -x$$

-:5.8:-

Find the equation to the straight line passing through (3, 4) and (5, 6)

SOLUTION

Equation to straight line passing through two points is

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Here $x_1 = 3, y_1 = 4, x_2 = 5, y_2 = 6$

$$(y-4)=\frac{6-4}{5-3}(x-3)$$

$$y-4=\frac{2}{2}(x-3)$$

$$y-4=x-3$$

$$y=x-3+4$$

$$y=x+1$$

-:5.9:-

Find the equation to the straight line passing through (1, 3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Here $x_1 = 1, y_1 = 3, x_2 = -7, y_2 = 8$

$$(y-3)=\frac{8-3}{-7-1}(x-1)$$

$$y-3=\frac{5}{-8}(x-1)$$

$$8(y-3)=-5(x-1)$$

$$8y-24=-5x+5$$

$$8y=-5x+5+24$$

$$8y=-5x+29$$

-:5.10:-

Find the equation to the straight line passing through (-1, 3) and (7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = -1$, $y_1 = 3$, $x_2 = 7$, $y_2 = 8$

$$(y-3) = \frac{8-3}{7+1}(x+1)$$

$$y-3 = \frac{5}{8}(x+1)$$

$$8(y-3) = 5(x+1)$$

$$8y-24 = 5x+5$$

$$8y = 5x+5+24$$

$$8y = 5x+29$$

-:5.11:-

Find the equation to the straight line passing through (-1, -3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = -1$, $y_1 = -3$, $x_2 = -7$, $y_2 = 8$

$$(y+3) = \frac{8-3}{-7+1}(x+1)$$

$$y+3 = \frac{11}{-6}(x-1)$$

$$-6(y+3) = 11(x+1)$$

$$-6y-18 = 11x+11$$

$$-6y = 11x+11+18$$

$$-6y = 11x+29$$

$$6y = -11x-29$$

-:5.12:-

Find the equation to the straight line passing through (0, –a) and (b, 0)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 0$, $y_1 = -a$, $x_2 = b$, $y_2 = 0$

$$(y + a) = \frac{0 + a}{b - 0}(x - 0)$$

$$y + a = \frac{a}{b}(x)$$

$$by + ab = ax$$

$$by = ax - ab$$

-:5.13:-

Find the equation to the straight line passing through (2, 7) and its slope is 3/5.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{3}{5}(x - 2)$$

$$5(y - 7) = 3(x - 2)$$

$$5y - 35 = 3x - 6$$

$$5y = 3x - 6 + 35$$

$$5y = 3x + 29$$

-:5.14:-

Find the equation to the straight line passing through (-2, -3) and its slope is -3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y + 3 = -3(x + 2)$$
$$y = -3x - 6 - 3$$
$$y = -3x - 9$$

-:5.15:-

Find the equation to the straight line passing through (-3, 4) and its slope is 2/3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{2}{3}(x + 3)$$
$$3(y - 4) = 2(x + 3)$$
$$3y - 12 = 2x + 6$$
$$3y = 2x + 6 + 12$$
$$3y = 2x + 8$$

-:5.16:-

Find the equation to the straight line passing through (2, -3) and its slope is -2.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y + 3 = -2(x - 2)$$
$$y + 3 = -2x + 4$$
$$y = -2x + 4 - 3$$
$$y = -2x + 1$$

-:5.17:-

Find the equation to the straight line passing through (0, 2) and its slope is -2/3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{2}{3}(x - 0)$$
$$3(y - 2) = -2x$$
$$3y - 6 = -2x$$
$$3y = -2x + 6$$

-:5.18:-

Find the equation to the straight line in which y-intercept is 2 and slope is 2/3.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = \frac{2}{3}x + 2 = \frac{2x + 6}{3}$$
$$3y = 2x + 6$$

-:5.19:-

Find the equation to the straight line in which y-intercept is -3 and slope is 3/4.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = \frac{3}{4}x - 3 = \frac{3x - 12}{4}$$
$$4y = 3x - 12$$

-:5.20:-

Find the equation to the straight line in which y-intercept is 5/2 and slope is -2/5.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = -\frac{2}{5}x + \frac{5}{2} = \frac{-4x + 25}{10} \Rightarrow 10y = -4x + 25$$

-:5.21:-

Find the equation to the straight line in which y-intercept is -4 and slope is -1.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = -1x - 4 \Rightarrow y = -x - 4$$

-:5.22:-

Find the slope and y-intercept of $3x + 2y - 7 = 0$

SOLUTION

$$3x + 2y - 7 = 0$$

$$2y = -3x + 7$$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = m = -3/2 and Intercept = c = 7/2

-:5.23:-

Find the slope and y-intercept of $3x + 2y + 7 = 0$

SOLUTION

$$3x + 2y + 7 = 0$$

$$2y = -3x - 7$$

$$y = -\frac{3}{2}x - \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = m = -3/2 and Intercept = c = -7/2

-:5.24:-

Find the slope and y-intercept of $5x - 4y + 8 = 0$

SOLUTION

$$5x - 4y + 8 = 0$$

$$-4y = -5x - 8 \Rightarrow 4y = 5x + 8$$

$$y = \frac{5}{4}x + 2 \Rightarrow y = mx + c$$

Hence Slope = m = 5/4 and Intercept = c = 2

-:5.25:-

Find the slope and y-intercept of 3x – 6y + 4 = 0

SOLUTION

$$3x - 6y + 4 = 0$$

$$-6y = -3x - 4$$

$$6y = 3x + 4$$

$$y = \frac{3}{6}x + \frac{4}{6}$$

$$y = \frac{1}{2}x + \frac{2}{3}$$

$$y = mx + c$$

Hence Slope = m = 1/2 and Intercept = c = 2/3

-:5.26:-

Find the slope of straight lines which passes through (3, 5) and (5, 3).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 3$, $y_1 = 5$, $x_2 = 5$ and $y_2 = 3$

$$m = \frac{3-5}{5-3} = \frac{-2}{2} = -1$$

-:5.27:-

Find the slope of straight lines which passes through (1, 4) and (-1, 4).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 1$, $y_1 = 4$, $x_2 = 1$ and $y_2 = 4$

$$m = \frac{4-4}{-1-1} = \frac{0}{-2} = 0$$

-:5.28:-

Find the slope of straight lines which passes through $\left(\frac{2}{3},\frac{3}{2}\right)$ and $\left(\frac{6}{7},\frac{7}{6}\right)$.

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = \frac{2}{3}, y_1 = \frac{3}{2}, x_2 = \frac{6}{7}$, and $y_2 = \frac{7}{6}$

$$m = \frac{\frac{7}{6} - \frac{3}{2}}{\frac{6}{7} - \frac{2}{3}} = \frac{\frac{7-9}{6}}{\frac{18-14}{21}}$$

$$\frac{\frac{-2}{6}}{\frac{4}{21}} = \frac{-2}{6} \times \frac{21}{4} = \frac{-42}{24} = \frac{-7}{4}$$

-:5.29-

Find the slope of straight lines which passes through $\left(\frac{1}{4},\frac{4}{7}\right)$ and $\left(\frac{4}{7},\frac{1}{4}\right)$.

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = \frac{1}{4}, y_1 = \frac{4}{4}, x_2 = \frac{4}{7}$, and $y_2 = \frac{1}{4}$

$$m = \frac{\frac{1}{4} - \frac{4}{7}}{\frac{4}{7} - \frac{1}{4}} = \frac{\frac{7-16}{28}}{\frac{16-7}{28}} \Rightarrow \frac{\frac{-9}{28}}{\frac{9}{28}} = \frac{-9}{28} \times \frac{28}{9} = -1$$

-:5.30:-

Graph the following linear function, y = 6x - 2

SOLUTION

Putting x = -2, -1, 0, 1, 2, 3

$y = 6(-2) - 2 = -12 - 2 = -14$

$y = 6(-1) - 2 = -6 - 2 = -8$

$y = 6(0) - 2 = 0 - 2 = -2$

$y = 6(1) - 2 = 6 - 2 = 4$

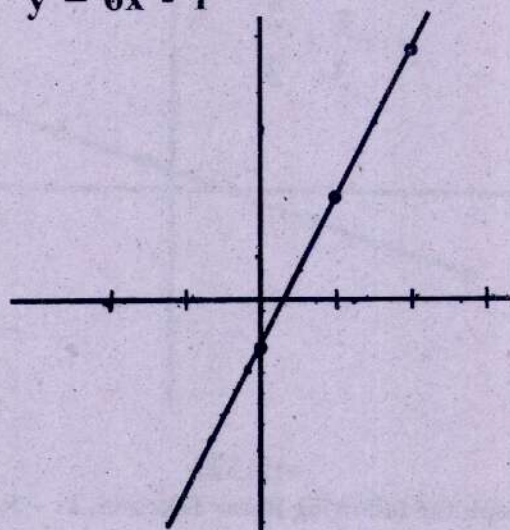
$y = 6(2) - 2 = 12 - 2 = 10$

$y = 6(3) - 2 = 18 - 2 = 16$

X	-2	-1	0	1	2	3
Y	-14	-8	-2	4	10	16

Graph

y = 6x - 1



-:5.31:-

Graph the following linear function, $y = \frac{x}{3} + \frac{2}{3}$

SOLUTION

$$y = \frac{x}{3} + \frac{2}{3}$$

Putting x = -11, -5, 1, 4, 7

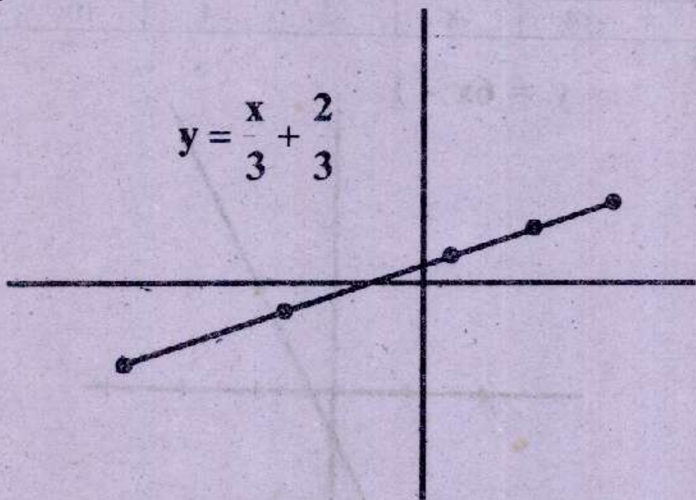
$$y = \frac{-11}{3} + \frac{2}{3} = \frac{-11+2}{3} = \frac{-9}{3} = -3, \quad y = \frac{-5}{3} + \frac{2}{3} = \frac{-5+2}{3} = \frac{-3}{3} = -1$$

$$y = \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1, \quad y = \frac{4}{3} + \frac{2}{3} = \frac{4+2}{3} = \frac{6}{3} = -3$$

$$y=\frac{7}{3}+\frac{2}{3}=\frac{7+2}{3}=\frac{9}{3}=3$$

x	-11	-5	1	4	7
y	-3	-1	1	2	3

Graph



-:5.32:-

Graph the following linear function, $2x-3y=12$

SOLUTION

$$2x-3y=12$$
$$-3y=12-2x$$
$$y=\frac{2x-12}{3}$$

Putting $x = 0, 3, 6, 9, 12$

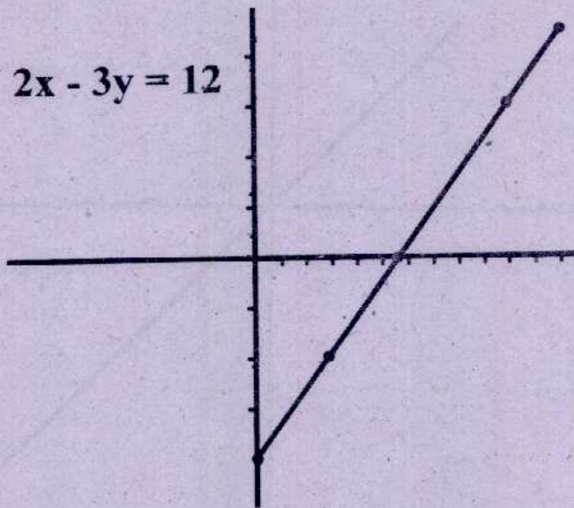
$$y=\frac{2(0)-12}{3}=\frac{-12}{3}=-4,\quad y=\frac{2(3)-12}{3}=\frac{6-12}{3}=\frac{-6}{3}=-2$$

$$y=\frac{2(6)-12}{3}=\frac{12-12}{3}=0,\quad y=\frac{2(9)-12}{3}=\frac{18-12}{3}=2$$

$$y=\frac{2(12)-12}{3}=\frac{24-12}{3}=4$$

X	0	3	6	9	12
Y	-4	-2	0	2	4

Graph



-:5.33:-

Graph the following linear function, $3x + 2y = 0$

SOLUTION

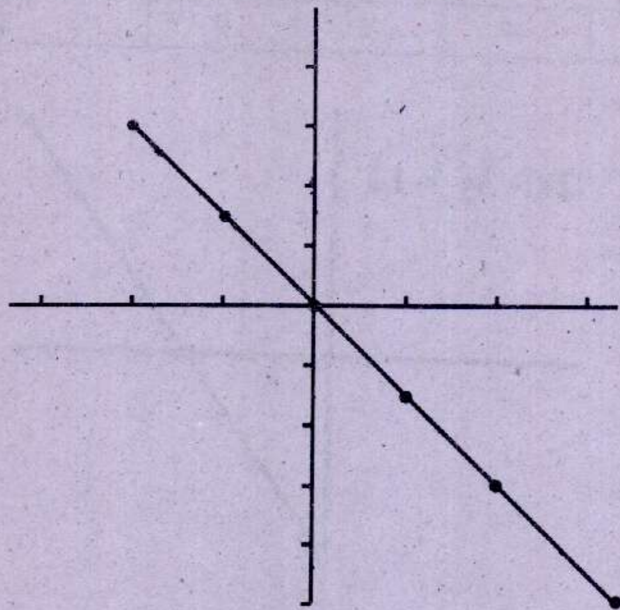
$$3x + 2y = 0$$
$$2y = -3x$$
$$y = \frac{-3x}{2}$$

Putting $x = -2, -1, 0, 1, 2, 3$

$$y = \frac{-3(-2)}{2} = \frac{6}{2} = 3,\ y = \frac{-3(-1)}{2} = \frac{3}{2} = 1.5,\ y = \frac{-3(0)}{2} = \frac{0}{2} = 0,$$
$$y = \frac{-3(1)}{2} = \frac{-3}{2} = -1.5,\ y = \frac{-3(2)}{2} = \frac{-6}{2} = -3,\ y = \frac{-3(3)}{2} = \frac{-9}{2} = -4.5$$

X	-2	-1	0	1	2	3
Y	3	1.5	0	-1.5	-3	-4.5

Graph



-:5.34:-

Graph the following linear function, 6x + 5y = 9

SOLUTION

$$6x + 5y = 9$$

$$5y = 9 - 6x$$

$$y = \frac{9-6x}{5}$$

Putting x = -1, 0, 1, 2, 3, 4

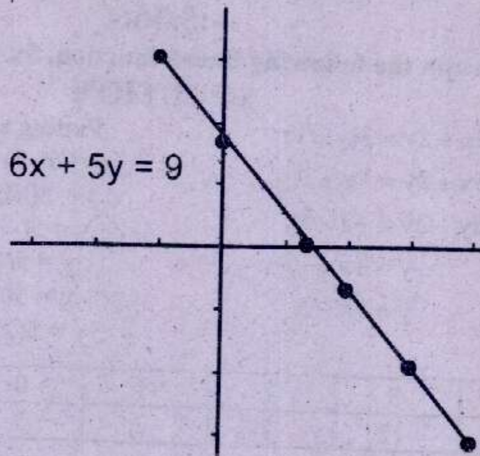
$$y = \frac{9-6(-1)}{5} = \frac{9+6}{5} = \frac{15}{5} = 3, \quad y = \frac{9-6(0)}{5} = \frac{9-0}{5} = 1.8$$

$$y = \frac{9-6(1)}{5} = \frac{9-6}{5} = \frac{3}{5} = 0.6, \quad y = \frac{9-6(2)}{5} = \frac{9-12}{5} = \frac{-3}{5} = -0.6$$

$$y = \frac{9-6(3)}{5} = \frac{9-18}{5} = \frac{-9}{5} = -1.8, \quad y = \frac{9-6(4)}{5} = \frac{9-24}{5} = \frac{-15}{5} = -3$$

X	-1	0	1	2	3	4
Y	3	1.8	0.6	-0.6	-1.8	-3

Graph



-:5.35:-

Graph the following linear function, $\frac{x}{2}+\frac{y}{4}=1$

SOLUTION

$$\frac{x}{2}+\frac{y}{4}=1$$

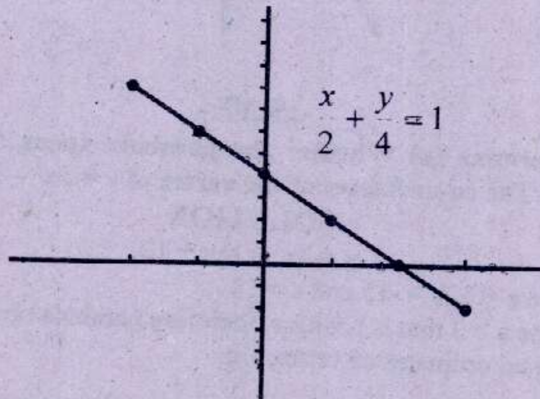
$$\frac{2x+y}{4}=1$$

$$2x+y=4$$

$y = 4 -2(-2) = 4 + 4 = 8$
$y = 4 -2(-1) = 4 + 2 = 6$
$y = 4 -2(0) = 4 + 0 = 4$
$y = 4 -2(1) = 4 - 2 = 2$
$y = 4 -2(2) = 4 - 4 = 0$
$y = 4 -2(-3) = 4 - 6 = -2$

X	-2	-1	0	1	2	3
Y	8	6	4	2	0	-2

Graph



-:5.36:-

Graph the following linear function, $5x + 2y = 3(y - 1)$

SOLUTION

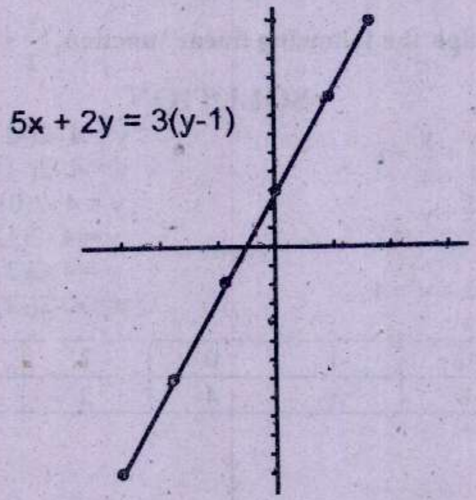
$$5x + 2y = 3(y - 1)$$
$$5x + 2y = 3y - 3$$
$$2y - 3y = -3 - 5x$$
$$-y = -5x - 3$$
$$y = 5x + 3$$

Putting x = -3, -2, -1, 0, 1, 2

$$y = 5(-3) + 3 = -15 + 3 = -12$$
$$y = 5(-2) + 3 = -10 + 3 = -7$$
$$y = 5(-1) + 3 = -5 + 3 = -2$$
$$y = 5(0) + 3 = 0 + 3 = 3$$
$$y = 5(1) + 3 = 5 + 3 = 8$$
$$y = 5(2) + 3 = 10 + 3 = 13$$

X	-3	-2	-1	0	1	2
Y	-12	-7	-2	3	8	13

Graph



-:5.37:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 3x^2 - 15x + 12$

SOLUTION

$$y = 3x^2 - 15x + 12$$

Here a = 3, b = -15 and c = 12

(a) Since a > 3 that is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-15)}{2(3)} = \frac{15}{6} = \frac{5}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(3)(12) - (15)^2}{4(3)}$$

$$= \frac{144 - 225}{12} = -\frac{81}{12} = -\frac{27}{4}$$

Hence Co-ordinate of vertex are x = 5/2 and y = -27/4

-:5.38:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 12x^2 - 2x - 4$

SOLUTION

$$y = 12x^2 - 2x - 4$$

Here a = 12, b = –2 and c = –4

(a) a = 12

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(12)} = \frac{2}{24} = \frac{1}{12}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(12)(-4) - (2)^2}{4(12)}$$

$$= \frac{-192 - 4}{24} = -\frac{196}{24} = -\frac{49}{6}$$

Hence Co-ordinate of vertex are x = 1/12 and y = -49/6

-:5.39:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = -2x^2 + 11x - 12$

SOLUTION

$$y = -2x^2 + 11x - 12$$

Here a = –2, b = 11 and c = –12

(a) a = –2

Since a<0 i.e. a is negative, therefore parabola opens downward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(11)}{2(-2)} = \frac{-11}{-4} = \frac{11}{4}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(-2)(-12) - (11)^2}{4(-2)}$$

$$= \frac{96 - 121}{-8} = \frac{-25}{-8} = \frac{25}{8}$$

Hence Co-ordinate of vertex are x = 11/4 and y = 25/8

-:5.40:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 - 9x + 18$

SOLUTION

$$y = x^2 - 9x + 18$$

Here a = 1, b = -9 and c = 18

(a) a = 1

Since a > 0 i.e. **a is** positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-9)}{2(1)} = \frac{9}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(18) - (-9)^2}{4(1)}$$

$$= \frac{72 - 81}{4} = -\frac{9}{4}$$

Hence Co-ordinate of vertex are x = 9/2 and y = -9/4

-:5.41:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 + 2x - 8$

SOLUTION

$$y = x^2 + 2x - 8$$

Here a = 1, b = 2 and c = –8

(a) a = 1

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(-8) - (2)^2}{4(1)}$$

$$= \frac{-32 - 4}{4} = \frac{36}{4} = -9$$

Hence Co-ordinate of vertex are x = -1 and y = -9

-:5.42:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 2x^2 - 12x + 18$

SOLUTION

$$y = 2x^2 - 12x + 18$$

Here a = 2, b = -12 and c = 18

(a) a = 2

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(2)(18) - (-12)^2}{4(2)}$$

$$= \frac{144 - 144}{8} = \frac{0}{8} = 0$$

Hence Co-ordinate of vertex are x = 3 and y = 0

-:5.43:-

Find the vertex and graph the following quadratic function:

$$\mathbf{y = 4x^2 - 3x + 1}$$

SOLUTION

$$y = 4x^2 - 3x + 1$$

Vertex is

$$\frac{-b}{2a} = \frac{-(-3)}{2(4)} = \frac{3}{8}, \quad f\left(\frac{-b}{2a}\right) = f\left(\frac{3}{8}\right)$$

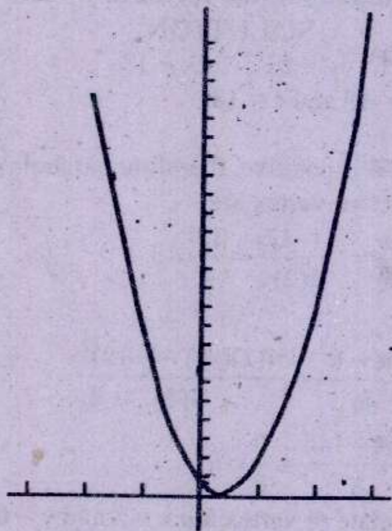
$$= 4\left(\frac{3}{8}\right)^2 - 3\left(\frac{3}{8}\right) + 1 = 4\left(\frac{9}{64}\right) - \frac{9}{8} + 1$$

$$= \frac{9}{16} - \frac{9}{8} + 1 = \frac{9-18+16}{16} = \frac{7}{16}$$

So the vertex is $\left(\frac{3}{8}, \frac{7}{16}\right)$

x	-2	-1	0	$^3/_8$	1	2	3
$4x^2$ $-3x$ $+1$	16 6 1	4 3 1	0 0 1	$^9/_{10}$ $^{-9}/_8$ 1	4 -3 1	16 -6 1	36 -9 1
$y = 4x^2-3x+1$	23	8	1	$^7/_{16}$	2	11	28

Graph



-:5.44:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - 2x$$

SOLUTION

$$y = x^2 - 2x$$

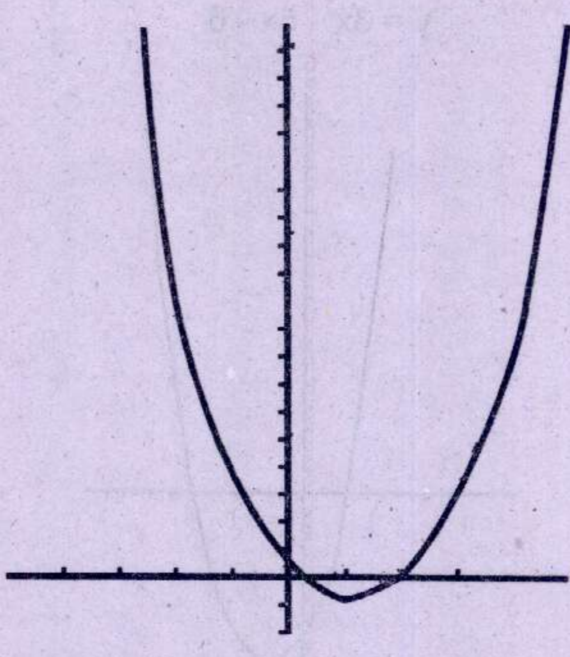
$$\frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1, \quad f\left(\frac{-b}{2a}\right) = f(1)$$

$$= (1)^2 - 2(1) = 1 - 2 = -1$$

Hence vertex is (1, -1)

x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
$-2x$	6	4	2	0	-2	-4	-6	-8	-10
$y = x^2-2x$	15	8	3	0	-1	0	3	8	15

Graph



-:5.45:-

Find the vertex and graph the following quadratic function:

$$y = 3x^2 - 5x - 6$$

SOLUTION

$$y = 3x^2 - 5x - 6$$

$$\frac{-b}{2a} = \frac{-(-5)}{2(3)} = \frac{5}{6},\ f\left(\frac{-b}{2a}\right) = f\left(\frac{5}{6}\right)$$

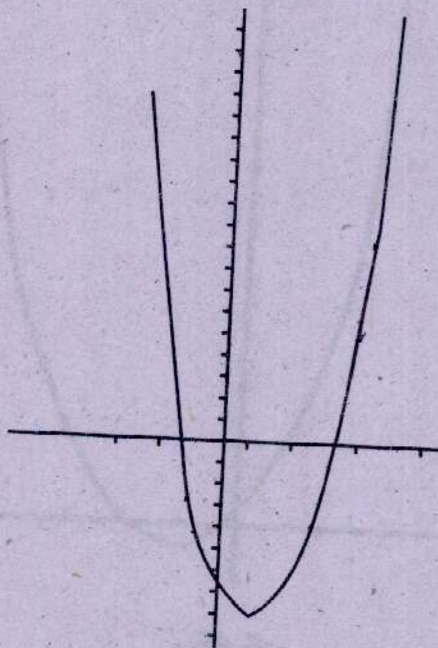
$$= 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 6 = \frac{25}{12} - \frac{25}{6} = -6$$

$$= \frac{25-50-72}{12} = -\frac{97}{12} = -8\frac{1}{12}$$

x	-2	-1	0	$^5/_6$	1	2	3	4
$3x^2$	12	3	0	$^{25}/_{12}$	3	12	27	48
$-5x$	10	5	0	$^{-25}/_6$	-5	-10	-15	-20
-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = 3x^2-5x-6$	16	2	-6	$^{-97}/_{12}$	-8	-4	6	22

Graph

$y = 3x^2 - 5x - 6$



-:5.46:-

Find the vertex and graph the following quadratic function:

$$y = 4x^2 - 4x + 1$$

SOLUTION

$$y = 4x^2 - 4x + 1$$

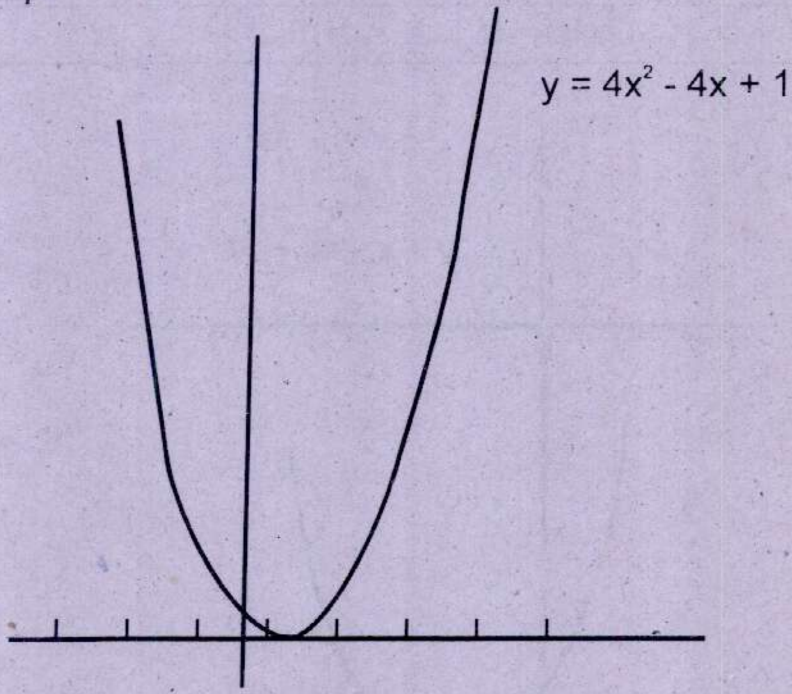
$$\frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{4}{8} = \frac{1}{2}$$

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) \Rightarrow = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 1 - 2 + 1 = 0$$

Hence vertex is (½, 0)

x	-2	-1	0	½	1	2	3	4
$4x^2$	16	4	0	1	4	16	36	64
$-4x$	8	4	0	-2	-4	-8	-12	-16
$+1$	1	1	1	1	1	1	1	1
$y = 4x^2-4x+1$	25	9	1	0	1	9	25	49

Graph



-:5.47:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - 4x - 17$$

SOLUTION

$$y = x^2 - 4x - 17$$

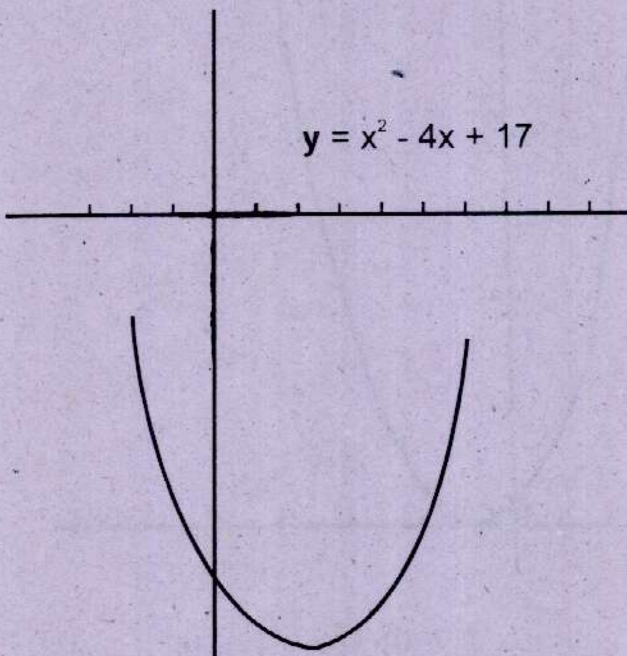
$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$$

$$f\left(\frac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) - 17 = 4 - 8 - 17 = -21$$

Hence vertex is (2, -21)

x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
$-4x$	8	4	0	-4	-8	-12	-16	-20	-24
-17	-17	-17	-17	-17	-17	-17	-17	-17	-17
$y = x^2-4x-17$	-5	-12	-17	-20	-21	-20	-17	-12	-5

Graph



-:5.48:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - x + 6$$

SOLUTION

$$y = x^2 - x + 6$$

$$\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$$

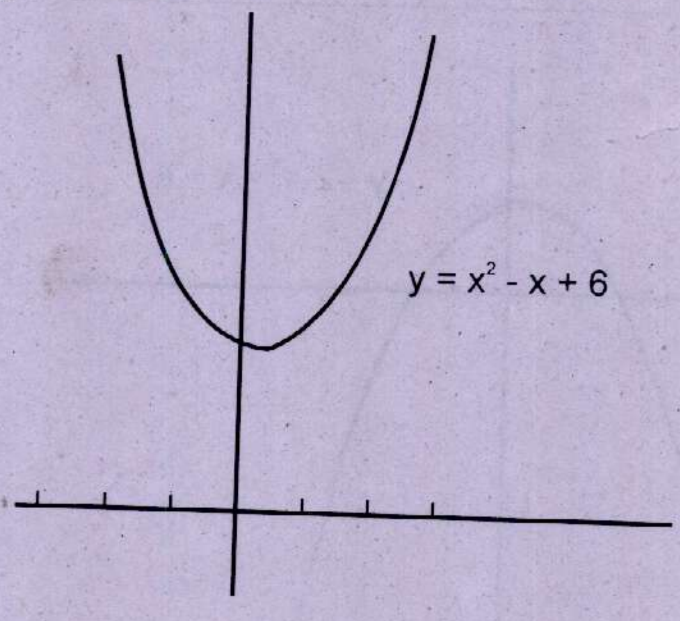
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 6$$

$$= \frac{1}{4} - \frac{1}{2} + 6 = \frac{23}{4} = 5\frac{3}{4}$$

$$\text{Vertex is } \left(\frac{1}{2}, \frac{23}{4}\right)$$

x	-2	-1	0	½	1	2	3
x^2	4	1	0	¼	1	4	9
-x	2	1	0	-½	-1	-2	-3
6	6	6	6	6	6	6	6
$y = x^2 - x + 6$	12	8	6	$^{23}/_4$	6	8	12

Graph



-:5.49:-

Find the vertex and graph the following quadratic function:

$$y = -2x^2 - x + 4$$

SOLUTION

$$y = -2x^2 - x + 4$$

$$\frac{-b}{2a} = \frac{(-1)}{2(2)} = \frac{1}{-4} = -\frac{1}{4}$$

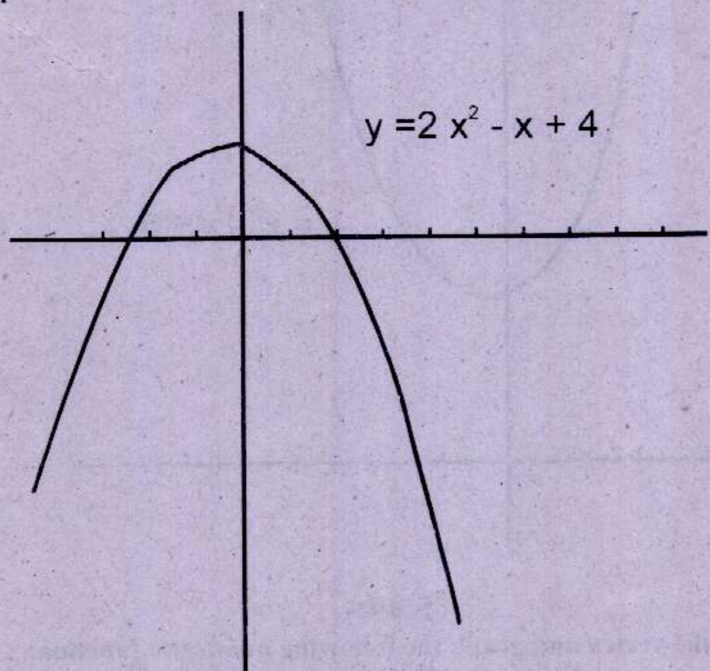
$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) + 4$$

$$= -\frac{1}{8} + \frac{1}{4} + 4 = \frac{1+2+32}{8} = \frac{33}{8} = 4\frac{1}{8}$$

Hence vertex is $\left(-\frac{1}{4}, \frac{33}{8}\right)$

x	-3	-2	-1	$^{-1}/_4$	0	1	2	3
$-2x^2$ $-x$ 4	-18 3 4	-8 2 4	-2 1 4	$^{-1}/_8$ ¼ 4	0 0 4	-2 -1 4	-8 -2 4	-18 -3 4
y = $2x^2-x+4$	-11	-2	3	$^{33}/_8$	4	1	-6	-17

Graph



-:5.50:-

Represent each of the following functions by a table and by a graph, letting x take on values within the limits indicated.

a)	$y = 2x - 3$	x between 0 and 8
b)	$3y = 6 - x$	x between 1 and 5
c)	$y = x^2 - 4x - 1$	x between -2 and 5
d)	$y = 16 + 10 - x^2$	x between a and b

SOLUTION

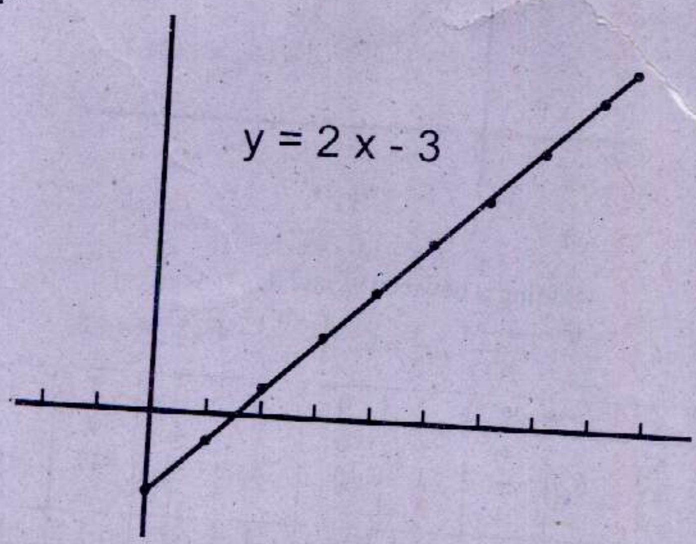
(a) $y = 2x - 3$

Putting x between 0 and 8

$$y = 2(0) - 3 = 0 - 3 = -3$$
$$y = 2(1) - 3 = 2 - 3 = -1$$
$$y = 2(2) - 3 = 4 - 3 = 1$$
$$y = 2(3) - 3 = 6 - 3 = 3$$
$$y = 2(4) - 3 = 8 - 3 = 5$$
$$y = 2(5) - 3 = 10 - 3 = 7$$
$$y = 2(6) - 3 = 12 - 3 = 9$$
$$y = 2(7) - 3 = 14 - 3 = 11$$
$$y = 2(8) - 3 = 16 - 3 = 13$$

x	0	1	2	3	4	5	6	7	8
y	-3	-1	1	3	5	7	9	11	13

Graph



(b)

$$3y = 6 - x$$

$$y = \frac{6-x}{3}$$

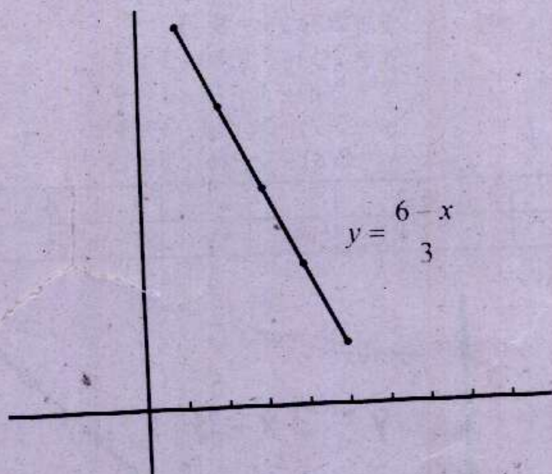
Putting x between 1 and 5

$$y = \frac{6-1}{3} = \frac{5}{3},\ y = \frac{6-2}{3} = \frac{4}{3},\ y = \frac{6-3}{3} = \frac{3}{3} = 1,$$

$$y = \frac{6-4}{3} = \frac{2}{3},\ y = \frac{6-5}{3} = \frac{1}{3}$$

X	1	2	3	4	5
y	$^5/_3$	$^4/_3$	1	$^2/_3$	$^1/_3$

Graph



(c)

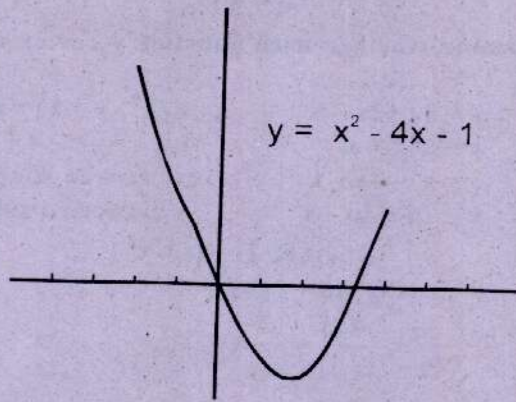
$$y = x^2 - 4x - 1$$

Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2,\quad f\left(\frac{-b}{2a}\right) = 4 - 8 - 1 = -5$$

x	-2	-1	0	1	2	3	4	5
x^2 $-4x$ -1	4 8 -1	1 4 -1	0 0 -1	1 4 -1	4 -8 -1	9 -12 -1	16 -16 -1	25 -20 -1
$y = x^2-4x-1$	11	4	-1	-4	-5	-4	-1	4

Graph



(d)

$$y = 16 + 10 - x^2$$

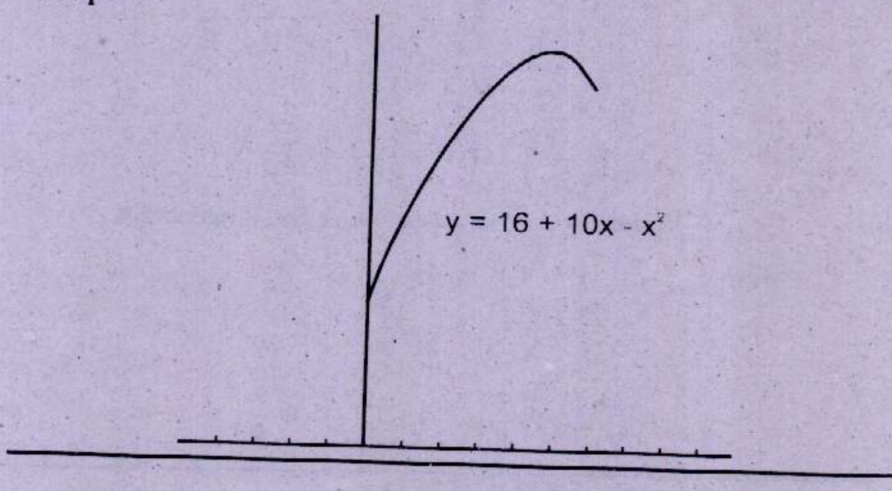
Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(10)}{2(-1)} = \frac{-10}{-2} = 5$$

$$f\left(\frac{-b}{2a}\right) = f(5) = 16 + 10(5) - (5)^2 = 16 + 50 - 25 = 41$$

x	0	1	2	3	4	5	6
16	16	16	16	16	16	16	16
10x	0	10	20	30	40	50	60
$-x^2$	0	-1	-4	-9	-16	-25	-36
$y = 16+10x-x^2$	16	25	32	37	40	41	40

Graph



-:5.51:-

Determine whether each function's vertex is minimum or maximum.

i)	$y = 8x + 2x - x^2$	**ii)**	$y = x^2 + 2x$
iii)	$y = x^2/2 + x$	**iv)**	$y = -4 + 7x - 4x^2$
c)	$y = x^2 - 4x - 1$		**x between -2 and 5**
d)	$y = 16 + 10 - x^2$		**x between a and b**

SOLUTION

(i)
$$y = 8 + 2x - x^2$$
$$\frac{-b}{2a} = \frac{-(2)}{2(-1)} = \frac{-2}{-2} = 1$$
$$f\left(\frac{-b}{2a}\right) = f(1) = 8 + 2(1) - (1)^2 = 8 + 2 - 1 = 9$$

Vertex is (1, 9), as a < 0 vertex is maximum

(ii)
$$y = x^2 + 2x$$
$$\frac{-b}{2a} = \frac{-(2)}{2(1)} = -1$$
$$f\left(\frac{-b}{2a}\right) = f(-1) = 1 + 2(-1)^2 = 1 - 2 = -1$$

Vertex is (-1, -1), as a > 0 vertex is maximum

(iii)
$$y = \frac{x^2}{2} + x$$
$$\frac{-b}{2a} = \frac{-1}{2(\frac{1}{2})} = \frac{-1}{1} = -1$$
$$f\left(\frac{-b}{2a}\right) = f(-1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

Vertex is (-1, -½), as a > 0, vertex is maximum

(iv)
$$y = -4 + 7x - 4x^2$$
$$\frac{-b}{2a} = \frac{-(7)}{2(-4)} = \frac{-7}{-8} = \frac{7}{8}$$
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{7}{8}\right) = -4 + 7\left(\frac{7}{8}\right) - 4\left(\frac{7}{8}\right)^2$$

$$f\left(\frac{-b}{2a}\right) = -4 + \frac{49}{8} - 4.\frac{49}{64} = -4 + \frac{49}{8} - \frac{49}{16}$$

$$= \frac{-64 + 98 - 49}{16} = -\frac{15}{16}$$

Hence vertex is $\left(\frac{7}{8}, \frac{-15}{10}\right)$. as $a < 0$ vertex is maximum

SET – B

-:5.1:-

The population size y of a certain city at time t is given by

$$y = f(t) = 4t^2 + 2t$$

(a) What is f(1), (b) What is f(2) and (c) What is f(3)

SOLUTION

$$y = f(t) = 4t^2 + 2t$$

(a) $f(1) = 4(1)^2 + 2(1) = 4 + 2 = 6$

(b) $f(2) = 4(2)^2 + 2(2) = 16 + 4 = 20$

(c) $f(3) = 4(3)^2 + 2(3) = 36 + 6 = 42$

-:5.2:-

A train traveled 60 miles the first hour and 50 miles each hour thereafter. Find the function which gives the distance covered in t hours. Also find the distance covered in one hour, 2 hour, 3 hours and 4 hours.

SOLUTION

Here total time = t hours

Speed of first hour = 60 miles

Speed of remaining (t – 1) hours = 50 miles each hour

So Function = $f(t) = 50(t - 1) + 60$

$f(1) = 50(1 - 1) + 60 = 50(0) + 60 = 60$ miles

$f(2) = 50(2 - 1) + 60 = 50 + 60 = 110$ miles

$f(3) = 50(3 - 1) + 60 = 100 + 60 = 160$ miles

$f(4) = 50(4 - 1) + 60 = 150 + 60 = 210$ miles

-:5.3:-

Find the quantity demanded from the following function when price (P) is 5, 10, 15, 20, 25. If $q_d = 5 + 2P$

SOLUTION

Here quantity demand function is

$$q_d = 5 + 2P$$

When P = 5 $\quad q_d = 5 + 2(5) = 5 + 10 = 15$
When P = 10 $\quad q_d = 5 + 2(10) = 5 + 20 = 25$
When P = 15 $\quad q_d = 5 + 2(15) = 5 + 30 = 35$
When P = 20 $\quad q_d = 5 + 2(20) = 5 + 40 = 45$
When P = 25 $\quad q_d = 5 + 2(25) = 5 + 50 = 55$

-:5.4:-

Find the quantity demanded from the following function when price (P) is 20, 40, 60, 80, 100. If $q_d = 300 - 3P$

SOLUTION

Here quantity demand function is

$$q_d = 300 - 3P$$

When P = 20 $\quad q_d = 300 - 3(20) = 300 - 60 = 240$
When P = 40 $\quad q_d = 300 - 3(40) = 300 - 120 = 180$
When P = 60 $\quad q_d = 300 - 3(60) = 300 - 180 = 120$
When P = 80 $\quad q_d = 300 - 3(80) = 300 - 240 = 60$
When P = 100 $\quad q_d = 300 - 3(100) = 300 - 300 = 0$

-:5.5:-

If the demand function and supply function for a specified per meter cloth are: $P + 2q = 100$ and $45P - 20q = 350$ respectively, compare the quantity demanded and quantity supplied when P = 14. Are there surplus cloth or not enough to meet demand.

SOLUTION

Here quantity demand function is

$$P + 2q = 100$$

Price = P = 14

The quantity demanded when P = 14 is

$$14 - 2q = 100$$
$$2q = 100 - 14$$
$$2q = 86$$
$$q = 43$$

The supply function is $45P - 20q = 350$

The quantity supplied when price = P = 14 is

$$45(14) - 20q = 350$$
$$630 - 20q = 350$$
$$-20q = 350 - 630$$
$$-20q = -280$$
$$q = 14$$

Cloth is no enough to meet the demand.

-:5.6:-

A factory determines that the overheads for producing a quantity of a certain items is Rs. 300 and the cost of each item is Rs. 20. Express the total expenses as a function of the number of items produced and compute the expenses for producing 12, 25, 50, 75 and 100 items.

SOLUTION

Let n represent the number of items produced. Then $20n + 300$ represent the total expenses. Let we use E to represent the expenses function, so that we have

$E(n) = 20n + 300$, where n is a whole number

We obtain:

$E(12) = 20(12 + 300 = 240 + 300 = 540$

$E(25) = 20(25) + 300 = 500 + 300 = 800$

$E(50) = 20(50) + 300 = 1000 + 300 = 1300$

$E(75) = 20(75) + 300 = 1500 + 300 = 1800$

$E(100) = 20(100) + 300 = 2000 + 300 = 2300$

-:5.7:-

Suppose that the cost function for producing certain items is given by $C(n) = 3n + 5$, where n represent the number of items produced. Compute C(150), C(500), C (750) and C(1500)

SOLUTION

Cost function is

$$C(n) = 3n + 5$$

When

$n = 150$ $\quad C(150) = 3(150) + 5 = 450 + 5 = \text{Rs. } 455$

$n = 500$ $\quad C(500) = 3(500) + 5 = 1500 + 5 = \text{Rs. } 1505$

$n = 750$ $\quad C(750) = 3(750) + 5 = 2250 + 5 = \text{Rs. } 2255$

$n = 1500$ $\quad C(1500) = 3(1500) + 5 = 4500 + 5 = \text{Rs. } 4505$

-:5.8:-

The profit function for selecting items is given by

$$\mathbf{P(n) = -n^2 + 500n + 61500}$$

Compute P(200), P(230), P(250) and P(260).

SOLUTION

Here

$$P(n) = -n^2 + 500n + 61500$$

$$\begin{aligned}
P(200) &= -(200)^2 + 500(200) + 61500 \\
&= -40000 + 100000 + 61500 \\
&= -40000 + 161500 = 121500 \\
P(230) &= -(230)^2 + 500(230) + 61500 \\
&= -52900 + 115000 + 61500 \\
&= -52900 + 176500 = 123600 \\
P(250) &= -(250)^2 + 500(250) + 61500 \\
&= -62500 + 125000 + 61500 \\
&= -62500 + 186500 = 124000 \\
P(260) &= -(260)^2 + 500(260) + 61500 \\
&= -67600 + 130000 + 61500 \\
&= -67600 + 191500 = 123900
\end{aligned}$$

-:5.9:-

The height of a projectile fired vertically into the air at an initial velocity of 64 feet per second is a function of the time (t) and is given by $\mathbf{h(t) = 64t - 16t^2}$

Compute h(1), h(2), h(3) and h(4)

SOLUTION

Here function of height $\quad h(t) = 64t - 16t^2$

When

$h(1) = 64(1) - 16(1)^2 = 64 - 16 = 48$ feet

$h(2) = 64(2) - 16(2)^2 = 128 - 16(4) = 128 - 64 = 64$ feet
$h(3) = 64(3) - 16(3)^2 = 192 - 16(9) = 192 - 144 = 48$ feet
$h(4) = 64(4) - 16(4)^2 = 256 - 16(16) = 256 - 256 = 0$ feet

-:5.10:-

A car rental agency charges Rs. 500 per day plus Rs. 1.25 a mile. Therefore, the daily charge for renting a car is a function of the number of miles travel (m) and can be expressed as:

$$C(m) = 500 + 1.25m$$

Compute C(75), C(100), C(150) and C(500)

SOLUTION

Here $C(m) = 500 + 1.25m$

When $C(75) = 500 + 1.25(75) = 500 + 93.75 = \text{Rs. } 593.75$
$C(100) = 500 + 1.25(100) = 500 + 125 = \text{Rs. } 625$
$C(150) = 500 + 1.25(150) = 500 + 187.50 = \text{Rs. } 687.50$
$C(500) = 500 + 1.25(500) = 500 + 625 = \text{Rs. } 1125$

-:5.11:-

A producer knows that he can sells as many items at Rs. 0.25 each as he can produce in a day. If the cost function is

$$C(x) = \text{Rs. } 0.20x + \text{Rs. } 70$$

Find the break even point.

SOLUTION

The amount received function for this problem would be $R(x) = \text{Rs. } 0.25x$. At break-even point the amount received must equal to the cost. Setting these two quantities equal gives:

$$R(x) = C(x)$$

$$0.25x = \text{Rs.} 20x + \text{Rs.} 70$$

$$0.25x - 0.20x = 70 \Rightarrow 0.05x = 70$$

$$x = 1400$$

Substituting this value into $R(x)=0.25x$

$$R(1400) = 0.25(1400) = 350$$

Hence break-even point is 1400, 350.

-:5.12:-

A firm knows that it can sells as many items at Rs. 1.25 each as it can produce in a day. If the cost function is C(x)=0.90x+Rs. 105. Find the break-even point.

SOLUTION

The amount received function for this problem is R(x)=Rs. 1.25x. At the break-even point, the amount received must equal to cost

$$R(x) = C(x)$$

$$Rs.1.25x = Rs.0.90x + Rs.105$$

$$1.25x - 0.90x = 105$$

$$0.35x = 105$$

$$x = \frac{105}{0.35} = \frac{10500}{35}$$

$$x = 300$$

Substituting this value into $R(x) = 1.25x$

$$R(300) = 1.25(300) = 375$$

Hence break-even point is 300, 375

-:5.13:-

The cost function for storing a particular item at XYZ corporation was found to be f(x) = 0.005x + 0.80 where x is the cost of the item. What is the cost of storing 84 items.

SOLUTION

Here $f(x) = 0.005x + 0.80$

$$f(84) = 0.005(84) + 0.80 = 0.42 + 0.80 = 1.22$$

-:5.14:-

Suppose a calculator has the total cost function C(x)=17+3400 and the revenue function R(x) = 34x

a) What is the equation of the profit function for the calculator?

b) What is the profit on 300 units?

SOLUTION

(a) $P(x) = R(x) - C(x)$

Profit Function = Revenue Function – Cost Function

$$= 34x - (17x + 3400)$$

$$= 34x - 17x - 3400$$

$$= 17x - 3400$$

(b) Profit on 300 units

$$P(300) = 17(300) - 3400$$

$$= 5100 - 3400 = Rs.\ 1700$$

EXERCISE NO. 6

SET – A

-:6.1:-

Solve for x, 5x + 4 = 19

SOLUTION

$$5x + 4 = 19$$

Add (-4) to both sides, we get

$$5x + 4(-4) = 19 + (-4)$$

$$5x = 15$$

Dividing both sides by 5

$$\frac{5x}{5} = \frac{15}{5}$$

$$x = 3$$

-:6.2:-

Solve for x, 2x + 8 = 24

SOLUTION

$$2x + 8 = 24$$

Add (-4) to both sides, we get

$$2x + 8 + (-8) = 24 + (-8)$$

$$2x = 16$$

Dividing both sides by 2

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

-:6.3:-

Solve for x, 7x – 3 = 4x + 21

SOLUTION

$$7x - 3 = 4x + 21$$

Subtract 4x from both sides to remove 4x from the right side

$$7x - 3 - 4x = 4x + 21 - 4x$$

$$7x - 4x - 3 = 21$$

Combine the like terms

$$3x - 3 = 21$$

Add 3 to both sides, we get

$$3x - 3 + 3 = 21 + 3$$

$$3x = 24$$

Dividing both sides by 3

$$\frac{32x}{3} = \frac{24}{3}$$

$$x = 8$$

-:6.4:-

Solve for y, 4(y-1)+5(y+2)=3(y-8)

SOLUTION

$$4(y-1) + 5(y+2) = 3(y-8)$$

$$4y - 4 + 5y + 10 = 3y - 24$$

Combine the like items

$$9y + 6 = 3y - 24$$

Subtract 3y from both sides,

$$9y + 6 - 3y = 3y - 24 - 3y$$

$$6y + 6 = -24$$

$$6y = -24 - 6$$

$$6y = -30$$

Dividing both sides by 6

$$\frac{6y}{6} = -\frac{30}{6}$$

$$y = -5$$

-:6.5:-

Solve for x, $x + 3(x-2) = 2x - 4$

SOLUTION

$$x + 3(x-2) = 2x - 4$$

$$x + 3x - 6 = 2x - 4$$

$$x + 3x - 2x = -4 + 6$$

$$4x - 2x = 2 \Rightarrow 2x = 2$$

$$x = 1$$

-:6.6:-

Solve for x, $4(2x - 5) = 3(2x + 18)$

SOLUTION

$$4(2x - 5) = 3(2x + 18)$$
$$8x - 20 = 6x + 54$$
$$8x - 6x = 54 + 20$$
$$2x = 74$$
$$x = 37$$

-:6.7:-

Solve for x, $3x - 2(x + 4) = 5x - 28$

SOLUTION

$$3x - 2(x + 4) = 5x - 28$$
$$3x - 2x - 8 = 5x - 28$$
$$3x - 2x - 5x = -28 + 8$$
$$3x - 7x = -28 + 8$$
$$-4x = -20$$
$$4x = 20 \Rightarrow x = 5$$

-:6.8:-

Solve for x, $2(x + 5) - 8(x - 6) = 10$

SOLUTION

$$2(x+5)-8(x-6)=10$$
$$2x+10-8x+48=10$$
$$2x-8x=10-10-48$$
$$-6x=-48$$
$$x=8$$

-:6.9:-

Solve for x, $-3(2x - 5) = 2(4x + 7)$

SOLUTION

$$-3(2x - 5) = 2(4x + 7)$$

$$6x + 15 = 8x + 14$$

$$-6x - 8x = 14 - 15$$

$$-14x = -1$$

$$14x = 1$$

$$x = \frac{1}{14}$$

-:6.10:-

Solve for x, $4(x - 2) - 3(x - 1) = 2(x + 6)$

SOLUTION

$$4(x - 2) - 3(x - 1) = 2(x + 6)$$

$$4x - 8 - 3x + 3 = 2x + 12$$

Combine the like terms

$$4x - 3x - 8 + 3 = 2x + 12$$

$$x - 5 = 2x + 12$$

$$x - 2x = 12 + 5$$

$$-x = 17$$

$$x = -17$$

-:6.11:-

Solve for x, $-(a - 1) - (3a - 2) = 6 + 2(a - 1)$

SOLUTION

$$-(a - 1) - (3a - 2) = 6 + 2(a - 1)$$

$$-a + 1 - 3a + 2 = 6 + 2a - 2$$

$$-a - 3a + 1 + 2 = 6 - 2 + 2a$$

$$-4a + 3 = 4 + 2a$$

$$-4a - 2a = 4 - 3$$

$$-6a = 1$$

$$a = -\frac{1}{6}$$

-:6.12:-

Solve for x, $3(x + 1) + x^2 = x^2 + 12$

SOLUTION

$$3(x + 1) + x^2 = x^2 + 12$$

$$3x + 3 + x^2 = x^2 + 12$$

$$3x + x^2 - x^2 = 12 - 3$$

$$3x = 9$$

$$x = 3$$

-:6.13:-

Solve for x, $8(6 + 4x) + 24 = -16 + 2(x + 7)$

SOLUTION

$$8(6 + 4x) + 24 = -16 + 2(x + 7)$$

$$48 + 32x + 24 = -16 + 2x + 14$$

$$32x - 2x = -16 + 14 - 48 - 24$$

$$30x = -74$$

$$x = -\frac{74}{30} = -\frac{37}{15}$$

-:6.14:-

Solve for x, $x - (x + 1) = 2x - (2x + 3)$

SOLUTION

$$x - (x + 1) = 2x - (2x + 3)$$

$$x - x - 1 = 2x - 2x - 3$$

$$x - x - 2x + 2x - 1 = -3$$

$$-1 = -3$$

There is no value of x which satisfies the equation. Therefore the equation is inconsistent.

-:6.15:-

Solve for x, 2.5x + 50 = 80

SOLUTION

$$2.5x + 50 = 80$$

Multiplying both sides by 10

$$10(2.5x) + 10(50) = 10(80)$$

$$25x + 500 = 800$$

$$25x = 800 - 500$$

$$25x = 300$$

$$x = 12$$

Note:- It is often easier to first clear the equation of all decimals by multiplying both sides by an appropriate power of 10.

-:6.16:-

Solve for x, 0.07x + 0.11x = 3.6

SOLUTION

$$0.07x + 0.11x = 3.6$$

Multiplying both sides by 100

$$100(0.07x + 0.11x) = 100(3.6)$$

$$7x + 11x = 360$$

$$18x = 360$$

$$x = 20$$

-:6.17:-

Solve for x, 0.21x + 0.11(700 – x) = 790

SOLUTION

$$0.21x + 0.11(700 - x) = 790$$

Multiplying both sides by 100

$$100[0.21x + 0.11(700 - x)] = 100(790)$$
$$21x + 11(700 - x) = 79000$$
$$21x + 7700 - 11x = 79000$$
$$21x - 11x = 79000 - 7700$$
$$10x = 71300$$
$$x = 7130$$

-:6.18:-

Solve for x, 0.09(x + 100) = 0.08x + 11

SOLUTION

$$0.09(x + 100) = 0.08x + 11$$

Multiplying both sides by 100

$$100[0.09(x + 100)] = 100(0.08x + 11)$$
$$9(x + 100) = 8x + 1100$$
$$9x + 900 = 8x + 1100$$
$$9x - 8x = 1100 - 900$$
$$x = 200$$

-:6.19:-

Solve for x, 0.8x + 0.9(850 – x) = 715

SOLUTION

$$0.8x + 0.9(850 - x) = 715$$

Multiplying both sides by 10

$$10[0.8x + 0.9(850 - x)] = 10(715$$
$$8x + 9(850 - x) = 7150$$
$$8x + 7650 - 9x = 7150$$
$$8x - 9x = 7150 - 7650$$
$$-x = -500$$
$$x = 500$$

-:6.20:-

Solve for x, $0.2(x + 0.2) + 0.5(x - 0.4) = 5.44$

SOLUTION

$$0.2(x + 0.2) + 0.5(x - 0.4) = 5.44$$

$$0.2x + 0.04 + 0.5x - 0.20 = 5.44$$

Multiplying both sides by 100

$$100(0.2x + 0.04 + 0.5x - 0.20) = 100(5.44)$$

$$100(0.7x + 0.16) = 544$$

$$70x + 16 = 544$$

$$70x = 544 - 16$$

$$70x = 528$$

$$x = \frac{528}{70} = \frac{264}{35}$$

-:6.21:-

Solve for x, $\frac{x-3}{2} = \frac{2x+4}{5}$

SOLUTION

$$\frac{x-3}{2} = \frac{2x+4}{5}$$

By cross multiplication, we get

$$5(x-3) = 2(2x+4)$$

$$5x - 15 = 4x + 8$$

$$5x - 4x = 8 + 15$$

$$x = 23$$

-:6.22:-

Solve for x, $\frac{x-2}{3} + \frac{x+1}{8} = \frac{5}{6}$

SOLUTION

$$\frac{x-2}{3}+\frac{x+1}{8}=\frac{5}{6}$$

Combine the left side's terms

$$\frac{8(x-2)+3(x+1)}{24}=\frac{5}{6}$$

$$\frac{8x-16+3x+3}{24}=\frac{5}{6}$$

$$\frac{11x-13}{24}=\frac{5}{6}$$

By cross multiplication, we get

$$6(11x-13)=5(24)$$

$$66x-78=120$$

$$66x=120+78$$

$$66x=198$$

$$x=\frac{198}{66}=3$$

-:6.23:-

Solve for y, $\frac{2y-3}{3}+\frac{y+1}{2}=3$

SOLUTION

$$\frac{2y-3}{3}+\frac{y+1}{2}=3$$

Combine the left side's terms

$$\frac{2(2y-3)+3(y+1)}{6}=3$$

$$\frac{4y-6+3y+3}{6}=3$$

$$\frac{7y-3}{6}=\frac{3}{1}$$

By cross multiplication, we get

$$7y - 3 = 18$$
$$7y = 18 + 3$$
$$7y = 21$$
$$y = 3$$

-:6.24:-

Solve for x, $\dfrac{2x+7}{9} - 4 = \dfrac{x-7}{12}$

SOLUTION

$$\frac{2x+7}{9} - 4 = \frac{x-7}{12}$$
$$\frac{2x+7}{9} - \frac{x-7}{12} = 4$$
$$\frac{4(2x+7) - 3(x-7)}{36} = 4$$
$$\frac{8x + 28 - 3x + 21}{36} = 4$$
$$\frac{5x+49}{36} = \frac{4}{1}$$

By cross multiplication

$$5x + 49 = 144$$
$$5x = 144 - 49$$
$$5x = 95$$
$$x = 19$$

-:6.25:-

Solve for x, $\dfrac{x-1}{4} - \dfrac{x-2}{6} = \dfrac{2}{3}$

SOLUTION

$$\frac{x-1}{4} - \frac{x-2}{6} = \frac{2}{3}$$

$$\frac{6(x-1)-4(x-2)}{24}=\frac{2}{3}$$

$$\frac{6x-6-4x+8}{24}=\frac{2}{3}$$

$$\frac{2x+2}{24}=\frac{2}{3}$$

By cross multiplication, we get

$$3(2x+2)=48$$

$$6x+6=48$$

$$6x=48-6$$

$$6x=42$$

$$x=7$$

-:6.26:-

Solve for x, $\frac{5}{6}(x+1)-\frac{2}{5}(x-1)=\frac{1}{2}$

SOLUTION

$$\frac{5}{6}(x+1)-\frac{2}{5}(x-1)=\frac{1}{2}$$

Combine the left side's terms

$$\frac{5(5x+5)-6(2x-2)}{30}=\frac{1}{2}$$

$$\frac{25x+25-12x+12}{30}=\frac{1}{2}$$

$$\frac{13x+37}{30}=\frac{1}{2}$$

By cross multiplication, we get

$$26x+74=30$$

$$26x=30-74$$

$$26x=-44$$

$$x=-\frac{44}{26}=-\frac{22}{13}$$

-:6.27:-

Solve for x, $\frac{1}{2}(3x+6)-\frac{1}{3}(2x-4)=20$

SOLUTION

$$\frac{1}{2}(3x+6)-\frac{1}{3}(2x-4)=20$$

$$\frac{3x+6}{2}-\frac{2x-4}{3}=20$$

Combine the left side's terms

$$\frac{3(3x+6)-2(2x-4)}{6}=20$$

$$\frac{9x+18-4x+8}{6}=20$$

By cross multiplication, we get

$$5x+26=6(20)$$

$$5x+26=120$$

$$5x=120-26$$

$$5x=94$$

$$x=\frac{94}{5}=18.8$$

-:6.28:-

Solve for x, $\frac{7x+8}{3x+1}=\frac{5}{3}$

SOLUTION

$$\frac{7x+8}{3x+1}=\frac{5}{3}$$

By cross multiplication, we get

$$3(7x+8)=5(3x+1)$$

$$21x+24=15x+5$$

$$21x-15x=5-24$$

$$6x=-19 \Rightarrow x=-\frac{19}{6}$$

-:6.29:-

Solve for x, $\dfrac{12x-6}{3}=\dfrac{4x+8}{4}$

SOLUTION

$$\frac{12x-6}{3}=\frac{4x+8}{4}$$

By cross multiplication, we get

$$4(12x-6)=3(4x+8)$$
$$48x-24=12x+24$$
$$48x-12x=24+24$$
$$36x=48$$
$$x=\frac{48}{36}=\frac{4}{3}$$

-:6.30:-

Solve for x, $\dfrac{3x}{2}-\dfrac{x}{2}=\dfrac{5(x-4)}{6}$

SOLUTION

$$\frac{3x}{2}-\frac{x}{2}=\frac{5(x-4)}{6}$$
$$\frac{3x-x}{2}=\frac{5x-20}{6}$$
$$\frac{2x}{2}=\frac{5x-20}{6}\Rightarrow x=\frac{5x-20}{6}$$

By cross multiplication, we get

$$6x=5x-20$$
$$6x-5x=-20$$
$$x=-20$$

-:6.31:-

Solve for x, $\dfrac{2x-5}{3}-\dfrac{3x-1}{4}=\dfrac{2}{3}$

SOLUTION

$$\frac{2x-5}{3}-\frac{3x-1}{4}=\frac{2}{3}$$

$$\frac{4(2x-5)-3(3x-1)}{12}=\frac{2}{3}$$

$$\frac{8x-20-9x+3}{12}=\frac{2}{3}$$

$$\frac{-x-17}{12}=\frac{2}{3}$$

By cross multiplication, we get

$$3(-x-17)=24$$

$$-3x-51=24$$

$$-3x=24+51$$

$$-3x=75$$

$$x=25$$

-:6.32:-

Solve for x, $\frac{1}{2}(x+3)-\frac{2}{5}(x-9)=x+9$

SOLUTION

$$\frac{1}{2}(x+3)-\frac{2}{5}(x-9)=x+9$$

$$\frac{x+3}{2}-\frac{2(x-9)}{5}=x+9$$

$$\frac{x+3}{2}-\frac{2x-18}{5}=x+9$$

$$\frac{5(x+3)-2(2x-18)}{10}=x+9$$

$$\frac{5x+15-4x+36}{10}=x+9$$

$$\frac{x+51}{10}=\frac{x+9}{1}$$

By cross multiplication, we get

$$x + 51 = 10(x + 9)$$
$$x + 51 = 10x + 90$$
$$x - 10x = 90 - 51$$
$$-9x = 39 \Rightarrow x = \frac{-39}{9} = \frac{-13}{3}$$

-:6.33:-

Solve for x, $\frac{x}{7} - \frac{1-x}{5} = 2 - \frac{3x}{5}$

SOLUTION

$$\frac{x}{7} - \frac{1-x}{5} = 2 - \frac{3x}{5}$$
$$\frac{5x+7x-7}{35} = \frac{10-3x}{5}$$
$$\frac{5x+7x-7}{35} = \frac{10-3x}{5}$$
$$\frac{12x-7}{35} = \frac{10-3x}{5}$$

By cross multiplication, we get

$$5(12x - 7) = 35(10 - 3x)$$
$$60x - 35 = 350 - 105x$$
$$60x + 105x = 350 + 35$$
$$165x = 385 \Rightarrow x = \frac{385}{165} = \frac{7}{3}$$

-:6.34:-

Solve for x, $2x + \frac{4-x}{2} = 3x - \frac{8(x-2)}{6}$

SOLUTION

$$2x + \frac{4-x}{2} = 3x - \frac{8(x-2)}{6}$$
$$\frac{4x+4-x}{2} = \frac{18x-8x+16}{6} \Rightarrow \frac{3x+4}{2} = \frac{10x+16}{6}$$

By cross multiplication, we get

$$6(3x + 4) = 2(10x + 16)$$

$$18x + 24 = 20x + 32$$

$$18x - 20x = 32 - 24$$

$$-2x = 8 \Rightarrow x = -4$$

-:6.35:-

Solve for x, $\dfrac{1-8x}{4+3x}=\dfrac{2+16x}{3-6x}$

SOLUTION

$$\frac{1-8x}{4+3x}=\frac{2+16x}{3-6x}$$

By cross multiplication, we get

$$(3-6x)(1-8x) = (4+3x)(2+16x)$$

$$3(1-8x) - 6x(1-8x) = 4(2+16x) + 3x(2+16x)$$

$$3-24x-6x+48x^2 = 8+64x+6x+48x^2$$

$$48x^2-48x^2-24x-6x-64x-6x = 8-3$$

$$-100x = 5 \Rightarrow x = \frac{-5}{100} = \frac{-1}{20}$$

-:6.36:-

Solve for x, $\dfrac{9x-7}{6x+5}=\dfrac{6x-3}{4x+2}$

SOLUTION

$$\frac{9x-7}{6x+5}=\frac{6x-3}{4x+2}$$

By cross multiplication, we get

$$(4x+2)(9x-7)=(6x+5)(6x-3)$$

$$4x(9x-7)+2(9x-7)=6x(6x-3)+5(6x-3)$$

$$36x^2-28x+18x-14-36x^2+18x-30x=-15$$

$$36x^2-36x^2-28x+18x+18x-30x=-15+14$$

$$36x-58x=-1$$

$$-22x=-1 \Rightarrow 22x=1 \Rightarrow x=\frac{1}{22}$$

-:6.37:-

Solve for x, $\frac{4}{1-x}=\frac{92}{6+2x}$

SOLUTION

$$\frac{4}{1-x}=\frac{92}{6+2x}$$

By cross multiplication, we get

$$4(6+2x)=92(1-x)$$

$$24+8x=92-92x$$

$$8x+92x=92-24$$

$$100x=68$$

$$x=\frac{68}{100}=0.68$$

-:6.38:-

Solve for x, $\frac{x+1}{3x}=\frac{1}{x}-\frac{1}{3}$

SOLUTION

$$\frac{x+1}{3x}=\frac{1}{x}-\frac{1}{3}$$

$$\frac{x+1}{3x}=\frac{3-x}{3x}$$

By cross multiplication, we get

$$3x(x+1)=3x(3-x)$$

$$3x^2+3x=9x-3x^2$$

$$3x^2+3x^2+3x-9x=0$$

$$6x^2-6x=0$$

$$6x(x-1)=0 \Rightarrow x-1=0$$

$$x=1$$

-:6.39:-

Solve for N, $\frac{N}{2}+\frac{N}{7}=N-5$

SOLUTION

$$\frac{N}{2}+\frac{N}{7}=N-5$$

$$\frac{7N+2N}{14}=N-5$$

$$\frac{9N}{14}=\frac{N-5}{1}$$

By cross multiplication, we get

$$9N = 14(N-5)$$
$$9N = 14N - 70$$
$$9N - 14N = -70$$
$$-5N = -70$$
$$5N = 70$$
$$N = 14$$

-:6.40:-

Solve for x, $\frac{3}{x+1}-\frac{4}{3x-4}=\frac{24}{3x^2-x-4}$

SOLUTION

$$\frac{3}{x+1}-\frac{4}{3x-4}=\frac{24}{3x^2-x-4}$$

$$\frac{3(3x-4)-4(x+1)}{(x+1)(3x-4)}=\frac{24}{3x^2-x-4}$$

$$\frac{9x-12-4x-4}{3x^2-x-4}=\frac{24}{3x^2-x-4}$$

$$\frac{5x-16}{3x^2-x-4}=\frac{24}{3x^2-x-4}$$

Multiplying both sides by $3x^2-x-4$, We get

$$5x - 16 = 24$$
$$5x = 24 + 16$$
$$5x = 40$$
$$x = 8$$

SET – B

-:6.1:-

If 54 is subtracted from three times a certain number, result is 36. Find the number

SOLUTION

Let x represent the number to be found. The sentence "if 54 is subtracted from three times a certain number, the result is 36" translates into the equation $3x - 54 = 36$. Solving this equation, we obtain

$$3x - 54 = 36$$

$$3x = 36 + 54$$

$$3x = 90$$

$$x = 30$$

-:6.2:-

If two is added to five times a certain number the result is the same as if 16 is subtracted from twice the number. Find the number.

SOLUTION

Let x represent the number to be found.

$$5x + 2 = 2x - 16$$

$$5x - 2x = -16 - 2$$

$$3x = -18$$

$$x = -6$$

-:6.3:-

The sum of three consecutive integers is 54, what are the numbers.

SOLUTION

Let x be the first number, then

$x + 1$ is the second number and

$x + 2$ is the third number

$$x + (x + 1) + (x + 2) = 54$$

$$3x + 3 = 54 \Rightarrow 3x = 54 - 3$$

$$3x = 51 \Rightarrow x = 17$$

Hence three consecutive integers are 17, 18 and 19

-:6.4:-

The sum of the three consecutive integers is 13 greater than twice the smallest of the three integers. Find the integers.

SOLUTION

Let the smallest integer is = x
Then the second integer = x + 1
and the third integer = x + 2

$$x + (x + 1) + (x + 2) = 2x + 13$$
$$3x + 3 = 2x + 13$$
$$3x - 2x = 13 - 3$$
$$x = 10$$

Three consecutive integers are 10, 11 and 12

To check our answer, we must determine whether or not they satisfy the condition. Since 10, 11 and 12 are consecutive integers whose sum is 33 and since twice the smallest plus 13 is also 33 as 2(10)+13=33 so our answer are correct.

-:6.5:-

Find three consecutive integers such that twice the sum of the fist two integers is 11 more than three times the largest.

SOLUTION

Let first integer = x
the second integer = x + 1
and the third integer = x + 2
The equation is as follows:

$$2[x + (x + 1)] = 3(x + 2) + 11$$
$$2[2x + 1) = 3x + 6 + 11$$
$$4x + 2 = 3x + 17$$
$$4x - 3x = 17 - 2$$
$$x = 15$$

Hence First integer = 15
Second integer = 16
Third integer = 17

-:6.6:-

Verify that for any three consecutive integers, the sum of the smallest and largest is equal to twice the middle integer.

SOLUTION

Let the smallest integer = x
the middle integer = x + 1
and the largest integer = x + 2
Sum of smallest and largest = Twice the middle

$$x + x + 2 = 2(x + 1)$$

$$2x + 2 = 2x + 2$$

Hence verified

-:6.7:-

Find four consecutive integers whose sum is 106.

SOLUTION

Let first integer = x
the second integer = x + 1
the third integer = x + 2
and the fourth integer = x + 3

$$x + (x + 1) + (x + 2) + (x + 3) = 106$$

$$4x + 6 = 106$$

$$4x = 106 - 6$$

$$4x = 100$$

$$x = 25$$

Hence four consecutive integers are 25, 26, 27 and 28

-:6.8:-

Find a number such that three-eights of the number minus one half of it is 14 less than three-fourths of the number.

SOLUTION

Let x represent the number to be founded, then according to condition

$$\frac{3}{8}x - \frac{1}{2}x = \frac{3}{4}x - 14 \Rightarrow \frac{3x}{8} - \frac{x}{2} = \frac{3x}{4} - 14$$

$$\frac{3x-4x}{8}=\frac{3x-56}{4}$$

$$-\frac{x}{8}=\frac{3x-56}{4}$$

By cross multiplication, we get

$$-4x=8(3x-56)$$

$$-4x=24x-448$$

$$-4x-24x=-448$$

$$-28x=-448$$

$$28x=448$$

$$x=\frac{448}{28}=16$$

The required number is 16.

-:6.9:-

Find the number such that five sixths of the number is 4 more than two-thirds of the number

SOLUTION

Let x represent the number to be founded, then

$$\frac{5}{6}x=\frac{2}{3}x+4$$

$$\frac{5}{6}x-\frac{2}{3}x=4$$

$$\frac{15x-12x}{18}=4\Rightarrow\frac{3x}{18}=4$$

$$\frac{x}{6}=4$$

By cross multiplication

$$x=24$$

Hence required number is 24

-:6.10:-

Three-fourths of a number plus tow-fifths of the number is 13 less than one-half of the number. Find the number.

SOLUTION

Let x represent the number to be founded, then

$$\frac{3}{4}x + \frac{2}{5}x = \frac{1}{2}x - 13$$

$$\frac{3x}{4} + \frac{2x}{5} = \frac{x}{2} - \frac{13}{1}$$

$$\frac{15x + 8x}{20} = \frac{x - 26}{2}$$

$$\frac{23x}{20} = \frac{x - 26}{2}$$

By cross multiplication, we get

$$46x = 20x - 520$$

$$46x - 20x = -520$$

$$26x = -520$$

$$x = -20$$

Hence required number is -20

-:6.11:-

The sum of two numbers is 148. The larger number is two less than five times the smaller number. Find the two numbers.

SOLUTION

Let the smaller number is = x

Then the larger number = 5x -2

Sum of smaller and larger number = 148

$$x + 5x - 2 = 148$$

$$6x = 148 + 2$$

$$6x = 150$$

$$x = 25$$

Hence smaller number = 25

And larger number $= 5x - 2 = 5(25) - 2 = 125 - 2 = 123$

-:6.12:-

The difference of two numbers is 33. The larger number is one more than three times the smaller number. Find the numbers.

SOLUTION

Let the smaller number = x

then the larger number = 3x + 1

Larger number minus smaller number = 33

$$(3x + 1) - x = 33$$

$$3x + 1 - x = 33$$

$$2x + 1 = 33 \Rightarrow 2x = 33 - 1$$

$$2x = 32 \Rightarrow x = 16$$

Hence smaller number = 16

Larger number = 48 + 1 = 49

-:6.13:-

Danial received a motercyle repair bill of Rs. 106. This include Rs. 23 for parts, Rs 22 per hour for labour and Rs. 6 for taxes. Find the number of hours of labour.

SOLUTION

Let x represent the number of hours of labour. The 22x represents the total charge for labour. Charge for parts plus charge for labour plus tax equal to the total bill as follows:

Parts + Labour + Tax = Total Bill

$$23 + 22x + 6 = 106$$

Solving this equation, we get

$$22x + 29 = 106$$

$$22x = 106 - 29$$

$$22x = 77$$

$$x = \frac{77}{22}$$

$$x = \frac{7}{2} = 3\frac{1}{2} \text{ hours}$$

-:6.14:-

In a class of 92 students, the number of females is one less than twice the number of males. How many females and how many males are there in the class?

SOLUTION

Total number of students = 92

Let the number of males = x

then the number of females = 2x − 1

$$x + (2x - 1) = 92$$

$$x + 2x - 1 = 92$$

$$3x = 92 + 1$$

$$3x = 93$$

$$x = 31$$

Hence Number of males = 31 and

Number of females = 61

-:6.15:-

A board 20 feet long is cut into two pieces such that the length of one piece is the two-thirds of the length of the other piece. Find

(a) The length of shorter piece

(b) The length of larger piece

SOLUTION

Let x length of one piece = x

then the length of the other piece = $\frac{2}{3}x$

Sum of shorter piece and larger piece is 20 feet, that is;

$$\frac{2}{3}x + x = 20$$

$$\frac{2x + 3x}{3} = 20$$

$$\frac{5x}{3} = \frac{20}{1}$$

By cross multiplication, we get

$$5x = 60$$

$$x = 12$$

Length of larger piece = x = 12 feet

Length of shorter piece = 2/3 x 12 = 8 feet

$\bar{x} = \frac{\Sigma x}{n}$

-:6.16:-

The average of the salaries of Ali, Asghar and Arshid is Rs. 24000. if Asghar earns Rs. 10000 more than Ali and Arshid's salary is Rs. 2000 more than twice of Ali's salary, find the salary of each person.

SOLUTION

If average salary of three persons = Rs. 24000
Then total salary of three persons = 3 x 24000 = Rs. 72000
Let the Ali's Salary = Rs. X
then Asghar's Salary = Rs. (x + 10000)
and Arshid's Salary = Rs. (2x + 2000)
Ali's Salary + Asghar's Salary + Arshid's Salary = Rs. 72000

$$x + (x + 10000) + (2x + 2000) = 72000$$
$$4x + 12000 = 72000$$
$$4x = 72000 - 12000$$
$$4x = 60000$$
$$x = \text{Rs. } 15000$$

Hence Ali's Salary = Rs. 15000
Asghar's Salary = Rs. 25000
Arshid's Salary = Rs. 32000

-:6.17:-

The sum of the present ages of Noor and his father is 64 years. After eight years Noor will be three-fifths as old as his father at that time. Find the present ages of Noor and his father.

SOLUTION

Sum of ages of Noor and his Father = 64 years
Let the present age of Noor = x
Then the present age of Father = 64 – x
After 8 years the age of Noor = x + 8
After 8 years the age of his Father = (64 – x) + 8
After 8 years Noor will be three-fifth as old as his father, that is

$$x + 8 = \frac{3}{5}[(64 - x) + 8] \Rightarrow x + 8 = \frac{3}{5}(72 - x)$$

$$\frac{x + 8}{1} = \frac{216 - 3x}{5}$$

By cross multiplication

$$5x + 40 = 216 - 3x$$
$$5x + 3x = 216 - 40$$
$$8x = 176$$
$$x = 22$$

Hence Present age of Noor = x = 22 years
and present age of his Father = 64–x = 64–22 = 42 years

-:6.18:-

Ahmed took three mathematics exams and had an average score of 88. his second exam was 10 points better than his first exam and his third exam was 4 points better than his second exam. What were his three exam scores?

SOLUTION

Average scores of three exams = 88 score
Total scores of three exams = 3 x 88 = 264
Let the scores of first exam = x scores
then the scores of second exam = x + 10
and the scores of third exam = (x + 10) + 4 = x + 14
Sum of three exams scores are

$$x + (x + 10) + (x + 14) = 264$$
$$3x + 24 = 264$$
$$3x = 264 - 24$$
$$3x = 240$$
$$x = 80$$

Hence First exam scores = 80
Second exam score = 90
Third exam score = 94

-:6.19:-

A bus is carrying 32 passengers, some with Rs. 3 tickets and the remainder with Rs. 5 tickets. If the total receipts from these passengers are Rs. 114. Find the number of Rs. 3 fares.

SOLUTION

Let x = Rs. 3 tickets passengers, then Rs. 5 tickets passengers are 32-x

$$3x+5(32-x)=114$$
$$3x+160-5x=114$$
$$3x-5x=114-160$$
$$-2x=-46$$
$$2x=46$$
$$x=23$$

So No. of passengers of Rs. 3 are 23.

-:6.20:-

Going from one town to another, a man drives his car at 35 miles an hour, and returning he drives at 25 miles an hour. The round trip takes 6 hours. Find the distance between the towns.

SOLUTION

Let x be the distance between two towns

Total time consumed = 6 hours

Distance = (Speed Rate) (Time)

$$\text{Time} = \frac{\text{Distance}}{\text{Speed Rate}}$$

The time used to drive from one town to other $= \frac{x}{35}$

The time used to drive back $= \frac{x}{25}$

Total time used to go and come back $= \frac{x}{35} + \frac{x}{25}$

$$\frac{x}{35} + \frac{x}{25} = 6$$
$$\frac{7x+5x}{175} = 6$$
$$\frac{12x}{175} = 6$$

By cross multiplication, we get

$$12x = 10.50$$
$$x = 87.5 \text{ miles}$$

The distance from between town is 87.5 miles.

-:6.21:-

A man who regularly drives between two cities, finds that if he drives at an average speed of 120 km/h, he arrives 2 hours before his usual time, and if he drives at an average speed of 60 km/h, he arrives 3 hours later than his usual time. What is the usual time.

SOLUTION

Let usual time to drive between two cities be x.

Average speed is 120 km/h, he arrive two hours before.

The consumed time at 120 km/h is (x-2) hours. The distance covered is

$$\text{Average Speed x Time} = 120(x-2)$$

Again

The average speed is 60 km/h, he arrive three hours late.

The time consumed at 60 km/h is (x+3)

The distance covered is 60 (x+3)

The distance between two cities will remain same.

$$120(x-2) = 60(x+3)$$

$$120x - 240 = 60x + 180$$

$$120x - 60x = 180 + 240$$

$$60x = 420$$

$$x = 7$$

-:6.22:-

A car covers 635 km by going for 4 hours at a certain speed, then for 3 hours at 5 km/h faster, and then for 2 hours at 5 km/h slower. Find the first speed.

SOLUTION

Total distance = 635 km

Let the first speed = x km/h

The time consumed with first speed is 4 hours.

Distance covered by first speed is

$$\text{Time x Speed} = 4x$$

The second time is 3 hours and second speed is 5 km/h faster, so second speed is (x+5) km/h

The distance covered is

$$\text{Time x Speed} = 3(x+5)$$

The third time is 2 hours and third speed is 5 km/h slower than the first speed, so third speed is $(x - 5)$ km/h

The distance covered is

$$\text{Time} \times \text{Speed} = 2(x - 5)$$

Hence total distance covered is

$$4x + 3(x + 5) + 2(x - 5) = 635$$

$$4x + 3x + 15 + 2x - 10 = 635$$

$$4x + 3x + 2x + 15 - 10 = 635$$

$$9x + 5 = 635 \Rightarrow 9x = 635 - 5$$

$$9x = 630$$

$$x = \frac{630}{9} = 70$$

The first speed is 70 km/h

-:6.23:-

A sum of Rs. 17000 is invested, part at 3% simple interest and the remainder at 4% simple interest. If the annual interest is Rs. 600, how much was invested at eaeh rate?

SOLUTION

Let x = The amount invested at 3%

17000-x = The amount invested at 5%

The income from the 3% investment = 0.03x and

The income from the 4% investment = 0.04(17000-x)

$$0.03x + 0.04(17000 - x) = \text{Rs. } 600$$

$$0.03x + 680 - 0.04x = \text{Rs. } 600$$

$$-0.01x + 680 = 600$$

$$-0.01x = 600 - 680 \Rightarrow -0.01x = -80$$

$$x = \text{Rs. } 8000$$

Amount of investment at 3% = Rs. 8000

Amount of investment at 4% = Rs. 9000

-:6.24:-

Mrs. B invested Rs. 30,000; a part at 5% and part at 8%. The total interest on the investment was Rs. 2100. How much did she invested at each rate?

SOLUTION

Let x = The amount of investment at 5%, then
30.000 – x = The amount of investment at 8%
The income from the 5% investment = 0.05x and
The income from the 8% investment = 0.08(30000 – x)

$$0.05x + 0.08(30000 - x) = 2100$$

$$0.05x + 2400 - 0.08x = 2100$$

$$0.05x - 0.08x = 2100 - 2400$$

$$-0.03x = -300$$

$$0.03x = 300$$

$$x = \frac{30000}{3} = \text{Rs.}10000$$

Amount invested at 5% = Rs. 10000
Amount invested at 8% = Rs. 20000

-:6.25:-

A sum of Rs. 2000 is split between two investments, one paying 7% interest and the other 8%. If return on the 8% investment exceeds that on the 7% investment by Rs. 40, how much is invested at each rate?

SOLUTION

Let the x = The amount of investment at 7%, and
2000 – x = The amount of investment at 8%
The income from the 7% investment = 0.07x, and
The income from the 8% investment = 0.08(2000 – x)
Since the return or income of 8% exceed Rs. 40, than income of 7%, so the equation is as follows:

$$0.07x + 40 = 0.08(2000 - x)$$

$$0.07x + 40 = 160 - 0.08x$$

$$0.07x + 0.08x = 160 - 40$$

$$0.15x = 120$$

$$x = \frac{120}{0.15} = \frac{12000}{15} = \text{Rs.}800$$

Hence Amount of investment at 7% = Rs. 800
Amount of investment at 8% = Rs. (2000 = 800) = Rs. 1200

-:6.26:-

One machine plows a field in 4 days, another does it in 6 days. How long does it take using both machines?

SOLUTION

Let x be the required number of days in which both machines plow the field.

$$\frac{x}{4} = \text{The part plow by first machine}$$

and $$\frac{x}{6} = \text{The part plow by second machine}$$

$$\frac{x}{4} + \frac{x}{6} = 1$$

$$\frac{3x+2x}{12} = 1 = \frac{5x}{12} = 1$$

$$5x = 12 =$$

$$x = \frac{12}{5} = 2.4 \text{ days}$$

-:6.27:-

If a class room has a length that is 5 feet less than 2 times of its width and its perimeter is 80 feet, find width and length of the class room.

SOLUTION

Let x = The width of class room, then 2x-5 = The length of class room.

Sum of four sides of the class room is the perimeter of classroom.

$$2x + 2(2x - 5) = 80$$

$$2x + 4x - 10 = 80$$

$$6x - 10 = 80$$

$$6x = 8 + 10$$

$$6x = 90 = x = \frac{90}{6} = 15$$

Thus, the width of the class room is 15 feet and length of the class room in 2x-5 = 30 - 5 = 25 feet.

EXERCISE NO. 7

SET – A

-:7.9:-

Solve the equation $x^2 + 7x + 12 = 0$

SOLUTION

$$x^2 + 7x + 12 = 0$$

Here a = 1, b = 7 and c = 12

We multiply 1 and 12 get 12.

Two factors of 12 with their sum = 7 are 4 and 3 as

$$4 \times 3 = 12$$
$$4 + 3 = 7$$

Hence

$$x^2 + 4x + 3x + 12 = 0$$
$$x(x + 4) + 3(x + 4) = 0$$
$$(x + 4)(x + 3) = 0$$

$x + 4 = 0$ or $x + 3 = 0$

$x = -4$ $x = -3$

-:7.10:-

Solve the equation $x^2 + 9x + 20 = 0$

SOLUTION

$$x^2 + 9x + 20 = 0$$

Here a = 1, b = 9 and c = 20

We multiply 1 and 20 get 20.

Two factors of 20 with their sum = 9 are 5 and 4 as

$$5 \times 4 = 20$$
$$5 + 4 = 9$$

Hence

$$x^2 + 5x + 4x + 20 = 0$$
$$x(x + 5) + 4(x + 5) = 0$$
$$(x + 5)(x + 4) = 0$$

$x + 5 = 0$ or $x + 4 = 0$

$x = -5$ $x = -4$

-:7.11:-

Solve the equation $2x^2 + 15x + 18 = 0$

SOLUTION

$$2x^2 + 15x + 18 = 0$$

Here a = 2, b = 15 and c = 18

We multiply 2 and 18 get 36.

Two factors of 36 with their sum = 15 are 12 and 3 as

$$12 \times 3 = 36$$
$$12 + 3 = 15$$

Hence

$$2x^2 + 12x + 3x + 18 = 0$$
$$2x(x+6) + 3(x+6) = 0$$
$$(x+6)(2x+3) = 0$$

$x + 6 = 0$ or $2x + 3 = 0$

$x = -6$ $\quad$ $x = -3/2$

-:7.12:-

Solve the equation $x^2 - 4x - 21 = 0$

SOLUTION

$$x^2 - 4x - 21 = 0$$

Here a = 1, b = -4 and c = -21

We multiply 1 and -21 get -21.

Two factors of -21 with their sum = -4 are -7 and 3 as

$$-7 \times 3 = -21$$
$$-7 + 3 = -4$$

Hence

$$x^2 - 7x + 3x - 21 = 0$$
$$x(x-7) + 3(x-7) = 0$$
$$(x-7)(x+3) = 0$$

$x - 7 = 0$ or $x + 3 = 0$

$x = 7$ $\quad$ $x = -3$

-:7.13:-

Solve the equation $10x^2 - 19x - 15 = 0$

SOLUTION

$$10x^2 - 19x - 15 = 0$$

Here $a = 10$, $b = -19$ and $c = -15$

We multiply 10 and -15 get -150.

Two factors of -150 with their sum = -19 are -25 and 6 as

$$-25 \times 6 = -150$$
$$-25 + 6 = 19$$

Hence

$$10x^2 - 25x + 6x - 15 = 0$$
$$5x(2x - 5) + 3(2x - 5) = 0$$
$$(2x - 5)(5x + 3) = 0$$
$$2x - 5 = 0 \quad \text{or} \quad 5x + 3 = 0$$
$$x = {}^{5}/_{2} \qquad\qquad x = -{}^{3}/_{5}$$

-:7.14:-

Solve the equation $x^2 + 10x = 18x - 15$

SOLUTION

$$x^2 + 10x = 18x - 15$$
$$x^2 + 10x - 18x + 15 = 0$$

Here $a = 1$, $b = -8$ and $c = 15$

We multiply 1 and 15 get 15.

Two factors of 15 with their sum = -8 are -5 and -3 as

$$-5 \times -3 = 15$$
$$5 + (-3) = -8$$

Hence

$$x^2 - 5x - 3x + 15 = 0$$
$$x(x - 5) - 3(x - 5) = 0$$
$$(x - 5)(x - 3) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 5 \qquad\qquad x = 3$$

-:7.15:-

Solve the equation $(x - 1)(x+5) = 7$

SOLUTION

$$(x - 1)(x + 5) = 7$$
$$x^2 + 5x - x - 5 = 7$$
$$x^2 + 4x - 5 - 7 = 0$$
$$x^2 + 4x - 12 = 0$$

Here a = 1, b = 4 and c = -12

We multiply 1 and -12 get -12.

Two factors of -12 with their sum = 4 are 6 and -2 as

$$6 \times -2 = -12$$

$$6 + (-2) = 4$$

Hence

$$x^2 + 6x - 2x + 12 = 0$$

$$x(x+6) - 2(x+6) = 0$$

$$(x+6)(x-2) = 0$$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ $\quad$ $x = 2$

-:7.16:-

Solve the equation $9x^2 - 16 = 0$

SOLUTION

$$9x^2 - 16 = 0$$

$$(3x)^2 - (4)^2 = 0$$

$$(3x-4)(3x+4) = 0$$

$3x - 4 = 0$ or $3x + 4 = 0$

$x = {}^4/_3$ $\quad$ $x = -{}^4/_3$

-:7.17:-

Solve the equation $21x^2 + 40x - 21 = 0$

SOLUTION

$$21x^2 + 40x - 21 = 0$$

Here a = 21, b = 40 and c = -21

We multiply 21 and -21, get -441.

Two factors of -441 with their sum = 40 are 49 and -9 as:

$$49 \times -9 = 441$$

$$49 - 9 = 40$$

$$21x^2 + 49x - 9x - 21 = 0$$

$$7x(3x+7) - 3(3x+7) = 0$$

$$(3x+7)(7x-3) = 0$$

$3x + 7 = 0$ or $7x - 3 = 0$

$x = -{}^7/_3$ $\quad$ $x = {}^3/_7$

-:7.18:-

Solve the equation $6x^2 + 17x - 14 = 0$

SOLUTION

$$6x^2 + 17x - 14 = 0$$

Sum of 17 and Product of 6 x – 14 = –84

We need to find two integers whose product is –84 and whose sum is 11. Obviously, 21 and –4 satisfy these conditions. We can proceed as follows:

$$6x^2 + 21x - 4x - 14 = 0$$
$$3x(2x + 7) - 2(2x + 7) = 0$$
$$(2x + 7)(3x - 2) = 0$$

$2x + 7 = 0$ or $3x - 2 = 0$

$2x = -7$ $\quad$ $3x = 2$

$x = -\dfrac{7}{2}$ $\quad$ $x = \dfrac{2}{3}$

Which gives the solution set {-7/2, 2/3}

-:7.19:-

Solve the equation $24n^2 - 38n + 15 = 0$

SOLUTION

$$24n^2 - 38n + 15 = 0$$

Sum of -38 and Product of 24 x 15 = –360

We need to find two integers whose product is –360 and whose sum is –38. Obviously, –20 and –18 satisfy these conditions. We can proceed as follows:

$$24n^2 - 20n - 18n + 15 = 0$$
$$4n(6n - 5) - 3(6n - 5) = 0$$
$$(6n - 5)(4n - 3) = 0$$

$6n - 5 = 0$ or $4n - 3 = 0$

$6n = 5$ $\quad$ $4n = 3$

$n = \dfrac{5}{6}$ $\quad$ $n = \dfrac{3}{4}$

Hence the solution set {5/6, 3/4}

-:7.20:-

Solve the equation $x^2 + 20x + 96 = 0$

SOLUTION

$$x^2 + 20x + 96 = 0$$

Sum of 20 and Product of 1 x 96 = 96

We need to find two integers whose product is 96 and whose sum is 20. Obviously, 12 and 8 satisfy these conditions. We can proceed as follows:

$$x^2 + 20x + 96 = 0$$

$$x^2 + 20x + 8x + 96 = 0$$

$$x(x + 12) + 8(x + 12) = 0$$

$$x + 12 = 0 \quad \text{or} \quad x + 8 = 0$$

$$x = -12 \qquad\qquad x = -8$$

Hence the solution set {−12, −8}

-:7.21:-

Solve the equation $20x^2 + 41x + 20 = 0$

SOLUTION

$$20x^2 + 41x + 20 = 0$$

Sum of 41 and Product of 20 x 20 = 400

We need to find two integers whose product is 400 and whose sum is 41. Obviously, 25 and 16 satisfy these conditions. We can proceed as follows:

$$20x^2 + 25x + 16x + 20 = 0$$

$$5x(4x + 5) + 4(4x + 5) = 0$$

$$(4x + 5)(5x + 4) = 0$$

$$4x + 5 = 0 \quad \text{or} \quad 5x + 4 = 0$$

$$4x = -5 \qquad\qquad 5x = -4$$

$$x = -\frac{5}{4} \qquad\qquad x = -\frac{4}{5}$$

Hence the solution set {−5/4, −4/5}

-:7.22:-

Solve the equation $(x + 8)(x - 6) = -24$

SOLUTION

$$(x+8)(x-6) = -24$$

$$x^2 - 6x + 8x - 48 + 24 = 0$$

$$x^2 + 2x - 24 = 0$$

Sum of 2 and Product of $1 \times -24 = -24$

We need to find two integers whose product is -24 and whose sum is 2. Obviously, 6 and -4 satisfy these conditions. We can proceed as follows:

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 6 \qquad\qquad x = 4$$

Hence the solution set $\{-6, 4\}$

-:7.23:-

Solve the equation $5x^2 = 43x - 24$

SOLUTION

$$5x^2 = 43x - 24$$

$$5x^2 - 43x + 24 = 0$$

Sum of -43 and Product of $5 \times 24 = 120$

We need to find two integers whose product is 120 and whose sum is -43. Obviously, -40 and -3 satisfy these conditions. We can proceed as follows:

$$5x^2 - 40x - 3x + 24 = 0$$

$$5x(x-8) - 3(x-8) = 0$$

$$(x-8)(5x-3) = 0$$

$$x - 8 = 0 \quad \text{or} \quad 5x - 3 = 0$$

$$x = 8 \qquad\qquad 5x = 3$$

$$x = \frac{3}{5}$$

Hence the solution set $\{8, 3/5\}$

-:7.24:-

Solve the equation $15y^2 + 34y + 15 = 0$

SOLUTION

$$15y^2 + 34y + 15 = 0$$

Sum of 34 and Product of 15 x 15 = 225

We need to find two integers whose product is 225 and whose sum is 34. Obviously, 25 and 9 satisfy these conditions. We can proceed as follows:

$$15y^2 + 25y + 9y + 15 = 0$$
$$5y(3y + 5) + 3(3y + 5) = 0$$
$$(3y + 5)(5y + 3) = 0$$

$$3y + 5 = 0 \quad \text{or} \quad 5y + 3 = 0$$
$$3y = -5 \qquad\qquad 5y = -3$$
$$y = -\frac{5}{3} \qquad\qquad y = -\frac{3}{5}$$

Hence the solution set {–5/3, –3/5}

-:7.25:-

Solve the equation $2x^2 + 3x - 8 = 0$

SOLUTION

$$2x^2 + 3x - 8 = 0$$
$$x^2 + \frac{3}{2}x - 4 = 0$$
$$x^2 + \frac{3}{2}x = 4$$

For completing the square of the left sides, we take one half of the co-efficient of x, and square the result. Add this number to both sides. In this example one half of $^3/_2$ is ¾ and its square is $(^3/_4)^2$. Add $(^3/_4)^2$ to both sides.

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 4 + \left(\frac{3}{4}\right)^2$$
$$\left(x + \frac{3}{4}\right)^2 = 4 + \frac{9}{16} = \frac{16+9}{16} = \frac{73}{16}$$

$$x+\frac{3}{4}=\pm\sqrt{\frac{73}{16}}=\pm\frac{\sqrt{73}}{4}$$

$$x=\pm\frac{\sqrt{73}}{4}-\frac{3}{4}$$

$$x=\frac{\sqrt{73}}{4}-\frac{3}{4}=\frac{\sqrt{73}-3}{4}$$

$$\text{or}\quad x=-\frac{\sqrt{73}}{4}-\frac{3}{4}=\frac{-\sqrt{73}-3}{4}$$

-:7.26:-

Solve the equation $15x^2-3-2=0$

SOLUTION

$$15x^2-x-2=0$$

$$x^2-\frac{x}{15}-\frac{2}{15}=0$$

$$x^2-\frac{x}{15}=\frac{2}{15}$$

For completing the square of the left sides, we take one half of the co-efficient of x, and square the result. Add this number to both sides. In this example one half of $^1/_{15}$ is $^1/_{30}$ and its square is $(^1/_{30})^2$. Add $(^1/_{30})^2$ to both sides.

$$x^2-\frac{x}{15}+\left(\frac{1}{30}\right)^2=\frac{2}{15}+\left(\frac{1}{30}\right)^2$$

$$\left(x-\frac{1}{30}\right)^2=\frac{2}{15}+\frac{1}{900}=\frac{120+1}{900}=\frac{121}{900}$$

Taking square root of both sides

$$x-\frac{1}{30}=\pm\frac{11}{30}$$

$$x=\frac{1}{30}\pm\frac{11}{30}$$

$$x=\frac{1}{30}+\frac{11}{30}=\frac{12}{30}=\frac{2}{5}$$

$$\text{or}\quad x=\frac{1}{30}-\frac{11}{30}=\frac{-10}{30}=\frac{-1}{3}$$

-:7.27:-

Solve the equation $x^2 - 6x + 8 = 0$

SOLUTION

$$x^2 - 6x + 8 = 0$$

$$x^2 - 6x = -8$$

In this example one half of 6 is 3 and its square is 9. Add 9 to both sides.

$$x^2 - 6x + 9 = -8 + 9$$

$$(x-3)^2 = 1$$

$$x - 3 = \pm 1$$

$$x - 3 = -1 \quad \text{or} \quad x - 3 = 1$$

$$x = 2 \qquad\qquad x = 4$$

-:7.28:-

Solve the equation $3x^2 - 9x + 5 = 0$

SOLUTION

$$3x^2 - 9x + 5 = 0$$

$$3x^2 - 9x = -5$$

$$x^2 - 3x = \frac{-5}{3}$$

In this example one half of 3 is $^3/_2$ and its square is $(^9/_4)$. Add $(^9/_4)$ to both sides.

$$x^2 - 3x + \frac{9}{4} = -\frac{5}{3} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{12}$$

Taking square root of both sides

$$x - \frac{3}{2} = \pm\frac{\sqrt{7}}{\sqrt{12}}, \quad x = \frac{3}{2} \pm \frac{\sqrt{7}}{\sqrt{12}}$$

$$x = \frac{\sqrt{7}}{2\sqrt{3}} + \frac{3}{2} = \frac{\sqrt{7} + 3\sqrt{3}}{2\sqrt{3}}$$

$$\text{or} \quad x = \frac{-\sqrt{7}}{2\sqrt{3}} + \frac{3}{2} = \frac{-\sqrt{7} + 3\sqrt{3}}{2\sqrt{3}}$$

-:7.29:-

Solve the equation $x^2 - 4x - 5 = 0$

SOLUTION

$$x^2 - 4x - 5 = 0$$

$$x^2 - 4x = 5$$

In this problems one half of 4 is 2 and its square is 4. Add 4 to both sides.

$$x^2 - 4x + 4 = 5 + 4$$

$$(x-2)^2 = 9$$

$$(x-2)^2 = (3)^2$$

$$x - 2 = \pm 3$$

$$x = 3+2 \quad \text{or} \quad x = -3+2$$

$$x = 5 \qquad\qquad x = -1$$

-:7.30:-

Solve the equation $x^2 - 10x + 7 = 0$

SOLUTION

$$x^2 - 10x + 7 = 0$$

$$x^2 - 10x = -7$$

In this problems one half of 10 is 5 and its square is 25. Add 25 to both sides.

$$x^2 - 10x + 25 = -7 + 25$$

$$(x-5)^2 = 18$$

$$x - 5 = \pm\sqrt{18} = \pm 3\sqrt{2}$$

$$x = 3\sqrt{2} + 5$$

$$\text{or} \quad x = -3\sqrt{2} + 5$$

-:7.31:-

Solve the equation $4x^2 - 123x = 0$

SOLUTION

$$4x^2 - 12x = 0$$

$$x^2 - 3x = 0$$

In this problem one half of 3 is $^3/_2$ and its square is $^9/_4$. Add $^9/_4$ to both sides.

$$x^2-3x+\frac{9}{4}=\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\left(\frac{3}{2}\right)^2=x-\frac{3}{2}=\pm\frac{3}{2}=\pm\frac{3}{2}+\frac{3}{2}$$

Taking square root of both sides

$$x=\frac{3}{2}+\frac{3}{2}=\frac{6}{2}=3$$

$$\text{or}\quad x=\frac{-3}{2}+\frac{3}{2}=0$$

-:7.32:-

Solve the equation $25x^2+30x+\frac{216}{25}=0$

SOLUTION

$$25x^2+30x+\frac{216}{25}=0\Rightarrow 25x^2+30x=-\frac{216}{25}$$

$$x^2+\frac{30}{25}x=-\frac{216}{25}\Rightarrow x^2+\frac{6}{5}x=-\frac{216}{625}$$

In this problem one half of $^6/_5$ is $^6/_{10}$ and its square is $(^6/_{10})^2$. Add $(^6/_{10})^2$ to both sides.

$$x^2+\frac{6}{5}x+\left(\frac{6}{10}\right)^2=-\frac{216}{625}+\left(\frac{6}{10}\right)^2$$

$$\left(x+\frac{6}{10}\right)^2=--\frac{216}{625}+\frac{36}{100}=\frac{36}{2500}\Rightarrow\left(x+\frac{6}{10}\right)^2=\left(\frac{6}{50}\right)^2$$

Taking square root of both sides

$$x+\frac{6}{10}=\pm\frac{6}{50}$$

$$x=\pm\frac{6}{50}-\frac{6}{10}$$

$$x=\frac{6}{50}-\frac{6}{10}=-\frac{24}{50}=-\frac{12}{25}$$

$$\text{or}\quad x=-\frac{6}{50}-\frac{6}{10}=-\frac{36}{50}=-\frac{18}{25}$$

-:7.33:-

Solve the equation by completing the square $x^2 - 2x - 8 = 0$

SOLUTION

$$x^2 - 2x - 8 = 0$$

$$x^2 - 2x = 8$$

In this problem one half of 2 is 1 and its square is 1. Add 1 to both sides

$$x^2 - 2x + 1 = 8 + 1$$

$$(x-1)^2 = 9$$

$$(x-1)^2 = (3)^2$$

Taking square root of both sides

$$x - 1 = \pm 3$$

$$x = \pm 3 + 1$$

$x = 3 + 1 = 4$ or $X = -3 + 1 = -2$

Hence solution set is {4, -2}

-:7.34:-

Solve the equation by completing the square $x^2 + 10x - 2 = 0$

SOLUTION

$$x^2 + 10x - 2 = 0$$

$$x^2 + 10x = 2$$

In this problem one half of 10 is 5 and its square is 25. Add 25 to both sides

$$x^2 + 10x + (5)^2 = 2 + 25$$

$$(x+5)^2 = 27$$

Taking square root of both sides

$$x + 5 = \pm\sqrt{27} = \pm 3\sqrt{3}$$

$$x = -5 \pm 3\sqrt{3}$$

$x = -5 + 3\sqrt{3}$ or $x = -5 - 3\sqrt{3}$

Hence solution set is $\{-5 + 3\sqrt{3}, -5 - 3\sqrt{3}\}$

-:7.35:-

Solve the equation by completing the square $2x^2 + 12x - 5 = 0$

SOLUTION

$$2x^2 + 12x - 5 = 0$$

$$2x^2 + 12x = 5$$

$$x^2 + 6x = \frac{5}{2}$$

In this problem one half of 6 is 3 and its square is 9. Add 9 to both sides

$$x^2 + 6x + (3)^2 = \frac{5}{2} + 9$$

$$(x+3)^2 = \frac{5+18}{2} = \frac{23}{2}$$

Taking square root of both sides

$$x + 3 = \pm\sqrt{\frac{23}{2}} = \sqrt{\frac{23\times 2}{2\times 2}} = \frac{\sqrt{46}}{2}$$

$$x = -3 \pm \frac{\sqrt{46}}{2}$$

$$x = -3 + \frac{\sqrt{46}}{2} \quad \text{or} \quad x = -3 - \frac{\sqrt{46}}{2}$$

Hence solution set is $\{-3 + \frac{\sqrt{46}}{2}, -3 - \frac{\sqrt{46}}{2}\}$

-:7.36:-

Solve the equation by completing the square $y^2 + 6y - 15 = 0$

SOLUTION

$$y^2 + 6y - 15 = 0$$

$$y^2 + 6y = 15$$

In this problem one half of 6 is 3 and its square is 9. Add 9 to both sides

$$y^2 + 6y + (3)^2 = 15 + 9 \Rightarrow (y+3)^2 = 24$$

Taking square root of both sides

$$y + 3 = \pm\sqrt{24} \Rightarrow y + 3 = \pm\sqrt{4\times 6} = \pm 2\sqrt{6}$$

$$y = -3 \pm 2\sqrt{6}$$

Hence solution set is $\{-3 + 2\sqrt{6}, -3 - 2\sqrt{6}\}$

-:7.37:-

Solve the equation by completing the square $x^2 + 5x - 14 = 0$

SOLUTION

$$x^2 + 5x - 14 = 0$$

$$x^2 + 5x = 14$$

In this problem one half of 5 is 5/2 and its square is $(5/2)^2 = 25/4$. Add 25/4 to both sides

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 14 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{56 + 25}{4} = \frac{81}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \left(\frac{9}{2}\right)^2$$

Taking square root of both sides

$$x + \frac{5}{2} = \pm\frac{9}{2} \Rightarrow x = -\frac{5}{2} \pm \frac{9}{2}$$

$$x = -\frac{5}{2} + \frac{9}{2} = \frac{-5+9}{2} = \frac{4}{2} = 2 \quad \text{or} \quad x = -\frac{5}{2} - \frac{9}{2} = \frac{-5-9}{2} = -\frac{14}{2} = -7$$

Hence solution set is $\{2, -7\}$

-:7.38:-

Solve the equation by completing the square $6x^2 - 13x = 28$

SOLUTION

$$6x^2 - 13x = 28 \Rightarrow x^2 - \frac{13}{6}x = \frac{28}{6}$$

In this problem one half of $-13/6$ is $-13/12$ and its square is $(-13/12)^2 = 169/144$. Add this square to both sides

$$x^2 + -\frac{13}{6}x + \left(-\frac{13}{12}\right)^2 = \frac{28}{6} + \frac{169}{144}$$

$$\left(x - \frac{13}{12}\right)^2 = \frac{672 + 169}{144} = \frac{841}{144}$$

$$\left(x - \frac{13}{12}\right)^2 = \left(\frac{29}{12}\right)^2$$

Taking square root of both sides

$$x-\frac{13}{12}=\pm\frac{29}{12} \Rightarrow x=\frac{13}{12}\pm\frac{29}{12}$$

$$x=\frac{13+29}{12}=\frac{42}{12}=\frac{7}{2} \quad \text{or} \quad x=\frac{13-29}{12}=-\frac{16}{12}=-\frac{4}{3}$$

Hence solution set is {7/2, –4/3}

-:7.39:-

Solve the equation $x^2+8x+15=0$

SOLUTION

$$x^2+8x+15=0$$

The equation is in general form

a = 1, b = 8 and c = 15

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-8\pm\sqrt{64-4(1)(15)}}{2(1)}=\frac{-8\pm\sqrt{64-60}}{2}$$

$$=\frac{-8\pm\sqrt{4}}{2}=\frac{-8\pm 2}{2}=\frac{-8+2}{2}=-\frac{6}{2}=-3$$

$$\text{or} \quad x=\frac{-8-2}{2}=-\frac{10}{2}=-5$$

-:7.40:-

Solve the equation $2x^2-7x-15=0$

SOLUTION

$$2x^2-7x-15=0$$

Here a = 2, b = -7 and c = -15

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(-15)}}{2(2)}=\frac{7\pm\sqrt{49+120}}{4}$$

$$=\frac{7\pm\sqrt{169}}{4}=\frac{7\pm 13}{4}=\frac{7+13}{4}=\frac{20}{4}=5$$

$$\text{or} \quad x=\frac{7-13}{4}=-\frac{6}{4}=-\frac{3}{2}$$

-:7.41:-

Solve the equation $6x^2 - 7x - 3 = 0$

SOLUTION

$$6x^2 - 7x - 3 = 0$$

Here a = 6, b = -7 and c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)} = \frac{7 \pm \sqrt{49+72}}{12}$$

$$= \frac{7 \pm \sqrt{121}}{12} = \frac{7 \pm 11}{12}$$

$$x = \frac{7+11}{12} = \frac{18}{12} = \frac{3}{2}$$

or $$x = \frac{7-11}{12} = -\frac{4}{12} = -\frac{1}{3}$$

-:7.42:-

Solve the equation $2x^2 + 7x - 1 = 0$

SOLUTION

$$2x^2 + 7x - 1 = 0$$

Here a = 2, b = 7 and c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)} = \frac{-7 \pm \sqrt{49+8}}{4} = \frac{-7 \pm \sqrt{57}}{4}$$

$$x = \frac{-7+\sqrt{57}}{4} \quad \text{or} \quad x = \frac{-7-\sqrt{57}}{4}$$

-:7.43:-

Solve the equation $x^2 - 12x + 36 = 0$

SOLUTION

$$x^2 - 12x + 36 = 0$$

Here a = 1, b = -12 and c = 36

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(36)}}{2(1)} = \frac{12 \pm \sqrt{144 - 144}}{2}$$

$$= \frac{12 \pm 0}{2} = 6$$

-:7.44:-

Solve the equation $12x^2 - 13x - 14 = 0$

SOLUTION

$$12x^2 - 13x - 14 = 0$$

Here a = 12, b = -13 and c = -14

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(12)(-14)}}{2(12)} = \frac{13 \pm \sqrt{169 + 672}}{24}$$

$$= \frac{13 \pm \sqrt{841}}{24} = \frac{13 \pm 29}{24}$$

$$x = \frac{13+29}{24} = \frac{42}{24} = \frac{7}{4} \quad \text{or} \quad x = \frac{13-29}{24} = -\frac{16}{24} = -\frac{4}{6} \quad \frac{2}{3}$$

-:7.45:-

Solve the equation $2x(3x - 5) = 3(7 - 8x)$

SOLUTION

$$2x(3x - 5) = 3(7 - 8x)$$

$$6x^2 - 10x = 21 - 24x$$

$$6x^2 - 10x + 24x - 21 = 0$$

$$6x^2 + 14x - 21 = 0$$

Here a = 6, b = 14 and c = -21

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-14 \pm \sqrt{(14)^2 - 4(6)(-21)}}{2(6)} = \frac{-14 \pm \sqrt{196 + 504}}{12}$$

$$x = \frac{-14 \pm \sqrt{700}}{12} = \frac{-14 \pm 10\sqrt{7}}{12} = \frac{-7 \pm 5\sqrt{7}}{6}$$

$$x = \frac{-7 + 5\sqrt{7}}{6} \quad \text{or} \quad x = \frac{-7 - 5\sqrt{7}}{6}$$

-:7.46:-

Solve the equation $x^2 + 2 = 9x$

SOLUTION

$$x^2 + 2 = 9x$$

$$x^2 - 9x + 2 = 0$$

Here $a = 1$, $b = -9$ and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(2)}}{2(1)} = \frac{9 \pm \sqrt{81 - 8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

$$x = \frac{9 + \sqrt{73}}{2} \quad \text{or} \quad x = \frac{9 - \sqrt{73}}{2}$$

-:7.47:-

Solve the equation by quadratic formula $2x^2 + 4x - 3 = 0$

SOLUTION

Here $a = 2$, $b = 4$ and $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16 + 24}}{4}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

Here Solution set is

$$\left\{\frac{-2 + \sqrt{10}}{2}, \frac{-2 - \sqrt{10}}{2}\right\}$$

-:7.48:-

Solve the equation by quadratic formula $2x^2 + 5x - 2 = 0$

SOLUTION

Here $a = 2$, $b = 5$ and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 - 4(2)(-2)}}{2(2)} = \frac{-5 \pm \sqrt{25 + 16}}{4} = \frac{-5 \pm \sqrt{41}}{4}$$

Here Solution set is

$$\left\{\frac{-5 + \sqrt{41}}{4}, \frac{-5 - \sqrt{41}}{4}\right\}$$

-:7.49:-

Solve the equation by quadratic formula $n(3n - 10) = 25$

SOLUTION

$$n(3n - 10) = 25$$

$$3n^2 - 10n = 25$$

$$3n^2 - 10n - 25 = 0$$

Here $a = 3$, $b = -10$ and $c = -25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3(-25)}}{2(3)} = \frac{10 \pm \sqrt{100 + 300}}{6}$$

$$= \frac{10 \pm \sqrt{400}}{6} = \frac{10 \pm 20}{6}$$

$$x = \frac{10 + 20}{6} = \frac{30}{6} = 5 \quad \text{or} \quad x = \frac{10 - 20}{6} = -\frac{10}{6} = -\frac{5}{3}$$

Here Solution set is {5, -5/3}

-:7.50:-

Solve the equation by quadratic formula $2x^2 - 17x + 30 = 0$

SOLUTION

$$2x^2 - 17x + 30 = 0$$

Here a = 2, b = −17 and c = 30

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-17) \pm \sqrt{(-17)^2 - 4(2)(30)}}{2(2)} = \frac{17 \pm \sqrt{289 - 240}}{4}$$

$$= \frac{17 \pm \sqrt{49}}{4} = \frac{17 \pm 7}{4}$$

$$x = \frac{17 + 7}{4} = \frac{24}{4} = 6 \quad \text{or} \quad x = \frac{17 - 7}{4} = -\frac{10}{4} = \frac{5}{2}$$

Here Solution set is {6, 5/2}

-:7.51:-

Solve the equation $3^{1+x} + 4.3^{-x} - 7 = 0$

SOLUTION

$$3^{1+x} + 4.3^{-x} - 7 = 0$$

$$3.3^x + 4.3^{-x} - 7 = 0$$

$$3.3^x + \frac{4}{3^x} - 7 = 0$$

Put $3^x = t$ $\quad 3t + \frac{4}{t} - 7 = 0$

Multiply t both sides

$$3t^2 + 4 - 7t = 0$$

$$3t^2 - 7t + 4 = 0$$

$$3t^2 - 3t - 4t + 4 = 0$$

$$3t(t-1) - 4(t-4) = 0$$

$$(3t-4)(t-1) = 0$$

$3t - 4 = 0$

$t = 4/3$ .... (i)

Putting value of t in (i)

$$3^x = \frac{4}{3}$$

$T - 1 = 0$

$t = 1$ .... (ii)

Putting value of t in (ii)

$$3^x = 1 \Rightarrow 3^x = 3^0 \quad \because 3^0 = 1$$

$$x = 0$$

Taking ln both sides

$x \ln 3 = \ln 4 - \ln 3$

$$x = \frac{\ln 4 - \ln 3}{\ln 3} = \frac{1.386 - 1.099}{1.099}$$

$$= \frac{0.287}{1.099} = 0.26$$

S.S. = {0, 0.26]

-:7.52:-

Solve the equation $2^{2x} - 3.2^{x+2} + 32 = 0$

SOLUTION

$$2^{2x} - 3.2^{x+2} + 32 = 0$$

$$(2^x)^2 - 3.2^x.4 + 32 = 0$$

$$(2^x)^2 - 12.2^x + 32 = 0$$

Put $2^x = t$

$$t^2 - 12t + 32 = 0$$

$$t^2 - 8t - 4t + 32 = 0$$

$$t(t-8) - 4(t-8) = 0$$

$$(t-4)(t-8) = 0$$

$t - 4 = 0$

$t = 4$ .... (i)

Putting value of t in (i)

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

$t - 8 = 0$

$t = 8$ .... (ii)

Putting value of t in (ii)

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

S.S. = {2, 3]

-:7.53:-

Solve the equation $2^x + 2^{-x+6} - 20 = 0$

SOLUTION

$$2^x + 2^{-x+6} - 20 = 0$$

$$2^x + 2^{-x}.2^6 - 20 = 0$$

$$2^x + 64.2^{-x} - 20 = 0$$

$$2^x + \frac{64}{2^x} - 20 = 0$$

Put $2^x = t$

$$t^2 + \frac{64}{t} - 20 = 0$$

$$t^2 + 64 - 20 = 0$$

$$t^2 - 20t + 64 = 0$$

$$t^2 - 16t - 4t + 64 = 0$$

$$t(t-16) - 4(t-16) = 0$$

$$(t-16)(t-4) = 0$$

$t - 4 = 0$

$t = 4$.......... (i)

Putting value of t in $2^x = t$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

$t - 16 = 0$

$t = 16$ .... (ii)

Putting value of t in $2^x = t$

$$2^x = 16$$

$$2^x = 2^4$$

$$x = 4$$

$$S.S. = \{2, 4\}$$

-:7.54:-

Solve the equation $3^{2x-1} - 12.3^x + 81 = 0$

SOLUTION

$$3^{2x-1} - 12.3^x + 81 = 0$$

$$3^{2x}.3^{-1} - 12.3^x + 81 = 0$$

$$\frac{3^{2x}}{3} - 12.3^x + 81 = 0$$

$$\frac{(3^x)^2}{3} - 12.3^x + 81 = 0$$

Put $3^x = t$ $\quad \frac{t^2}{3} - 12t + 81 = 0$

Multiply 3 both sides

$$t^2 - 36t + 243 = 0$$

$$t^2 - 27t - 9t + 243 = 0$$

$$t(t-27)-9(t-27)=0$$
$$(t-9)(t-27)=0$$

$t-9=0$

$t=9$.......... (i)

Putting value of t in $3^x = t$

$$3^x=9$$
$$3^x=3^2$$
$$x=2$$

$t-27=0$

$t=27$ .... (ii)

Putting value of t in $3^x = t$

$$3^x=27$$
$$3^x=3^3$$
$$x=3$$

$$S.S.=\{2, 3\}$$

-:7.55:-

Solve the equation $2\sqrt{x} = x - 8$

SOLUTION

$$2\sqrt{x}=x-8$$

Taking the square of both sides

$$(2\sqrt{x})^2=(x-8)^2$$
$$4x=x^2-16x+64$$
$$-x^2+4x+16x-64=0$$
$$-x^2+20x-64=0$$
$$x^2-20x+64=0$$

Sum of -20 and Product of $1 \times 64 = 64$

$$x^2-16x-4x+64=0$$
$$x(x-16)-4(x-16)=0$$
$$(x-16)(x-4)=0$$

$x-16=0$ or $x-4=0$

$x=16$ $x=4$

<u>Check</u>

When $x = 16$

$$2\sqrt{x}=8-x$$
$$2\sqrt{16}=16-8$$
$$2\times 4=16-8$$
$$8=8$$

When $x = 4$

$$2\sqrt{x}=8-x$$
$$2\sqrt{4}=4-8$$
$$2\times 2=-4$$
$$4\neq -4$$

Hence Solution of this problem is {16} only.

-:7.56:-

Solve the equation $\sqrt{5x+4}-\sqrt{3x+1}=1$

SOLUTION

$$\sqrt{5x+4}-\sqrt{3x+1}=1$$

$$\sqrt{5x+4}=1+\sqrt{3x+1}$$

Taking the square of both sides

$$5x+4=(1+\sqrt{3x+1})^2$$

$$5x+4=1+2\sqrt{3x+1}+3x+1$$

$$5x+4-1-3x-1=2\sqrt{3x+1}$$

$$5x-3x+2=2\sqrt{3x+1}$$

$$2x+2=2\sqrt{3x+1}\Rightarrow x+1=\sqrt{3x+1}$$

Taking square of both sides, we

$$(x+1)^2=(\sqrt{3x+1})^2$$

$$x^2+2x+1=3x+1$$

$$x^2+2x-3x+1-1=0$$

$$x^2-x=0$$

$$x(x-1)=0$$

$x=0$ Or $x-1=0\Rightarrow x=1$

Hence Solution set is $\{0, 1\}$

-:7.57:-

Find the discriminant of each of the following quadratic equations and determine whether the equation has (1) no real solution (2) one real solution (3) two real solution

SOLUTION

Equation	Discriminant	Nature of roots
(i) $x^2-3x+7=0$	$b^2-4ac=(-3)^2-4(1)(7)$ $=9-28=-19$	No real solution
(ii) $9x^2-12x+4=0$	$b^2-4ac=(-12)^2-4(9)(14)$ $=144-144=0$	On real solution

Equation	Discriminant	Nature of roots
(iii) $2x^2+5x-3=0$	$b^2-4ac=(5)^2-4(2)(-3)$ $=25+24=49$	Two real solution
(iv) $x^2+4x-12=0$	$b^2-4ac=(4)^2-4(1)(-12)$ $=16+48=64$	Two real solution
(v) $x^2-6x+11=0$	$b^2-4ac=(-6)^2-4(1)(11)$ $=36-44=-8$	No real solution
(vi) $3x^2+4x-2=0$	$b^2-4ac=(4)^2-4(3)(-2)$ $=16+24=40$	Two real solution
(vii) $15x^2+7x-2=0$	$b^2-4ac=(7)^2-4(15)(-2)$ $=49+120=169$	Two real solution
(viii) $4x^2+4x+1=0$	$b^2-4ac=(4)^2-4(4)(1)$ $=16-16=0$	One real solution

-:7.58:-

Find the discriminant of each of the following quadratic equations and determine whether the equation has (1) two complex but no real solution (2) two equal real solution or (3) two unequal real solution

SOLUTION

Equation	Discriminant	Nature of roots
(i) $x^2-3x+9=0$	$b^2-4ac=(-3)^2-4(1)(9)$ $=9-36=-27$	Two complex but non real solutions
(ii) $4x^2-20x+25=0$	$b^2-4ac=(-20)^2-4(4)(25)$ $=400-400=0$	Two equal real solutions
(iii) $3x^2+2x-1=0$	$b^2-4ac=(2)^2-4(3)(-1)$ $=4+12=16$	Two unequal real solutions
(iv) $x^2+6x-7=0$	$b^2-4ac=(6)^2-4(1)(-7)$ $=36+28=64$	Two unequal real solutions
(v) $5x^2+3x-9=0$	$b^2-4ac=(3)^2-4(5)(-9)$ $=9+180=189$	Two unequal real solutions
(vi) $16x^2+40x+25=0$	$b^2-4ac=(-40)^2-4(16)(25)$ $=1600-1600=0$	Two equal real solution

SET – B

-:7.1:-

Find two consecutive integers whose product is 56

SOLUTION

Let The first number = x, then

The second number = x + 1

According to given condition

$$x(x+1)=56$$
$$x^2+x=56$$
$$x^2+x-56=0$$
$$x^2+8x-7x-56=0$$
$$x(x+8)-7(x+8)=0$$
$$(x+8)(x-7)=0$$

$x+8=0$ or $x-7=0$

$x=-8$ $\quad$ $x=7$

When $x=-8$

$x+1=-8+1=-7$

When $x=7$

$x+1=7+1=8$

The integers are (7, 8) or (-8, -7)

-:7.2:-

Find two consecutive integers whose product is 91 such that one of the integers is one less than the twice the other number.

SOLUTION

Let the one number is x, then the other number is $2x-1$

Product of two such integers = $x(2x-1)$

$$x(2x-1)=91$$
$$2x^2-x=91$$
$$2x^2-x-91=0$$
$$2x^2-14x+13x-91=0$$
$$2x(x-7)+13(x-7)=0$$
$$(x-7)(2x+13)=0$$

$$x - 7 = 0 \quad \text{or} \quad 2x + 13 = 0$$
$$x = 7 \qquad 2x = -13$$
$$x = -\frac{13}{2}$$

If One integer = x = 7, then
Second integer = 2x − 1 = 14 − 1 = 13

-:7.3:-

Find the numbers whose product is 150 such that the one of the numbers is one more than four times the other numbers.

SOLUTION

Let The first number = x, then
The second number = 4x + 1

$$x(4x+1) = 150$$
$$4x^2 + x = 150$$
$$4x^2 + x - 150 = 0$$

Here $a = 4, b = 1, c = -150$
$a \times c = 4 \times -150 = -600$

$$4x^2 + 25x - 24x - 150 = 0$$
$$x(4x+25) - 6(4x+25) = 0$$
$$(4x+25)(x-6) = 0$$

$$4x + 25 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = -\frac{25}{4} \qquad x = 6$$

When $x = 6$
Then 4x + 1 = 24 + 1 = 25
Hence the required numbers are 6 and 25

-:7.4:-

Find two positive numbers having a sum 23 and a product of 126

SOLUTION

Let The one number is x
Since the sum of two positive number is 23
Then the second number = 23 − x

$$x(23 - x) = 126$$
$$23x - x^2 = 126$$
$$23x - x^2 - 126 = 0$$
$$-x^2 + 23x - 126 = 0$$
$$x^2 - 23x + 126 = 0$$
$$x^2 - 14x - 9x + 126 = 0$$
$$x(x - 14) - 9(x - 14) = 0$$
$$(x - 14)(x - 9) = 0$$

$x - 14 = 0$ or $x - 9 = 0$

$x = 14$ $\quad$ $x = 9$

When $x = 14$, $23 - x = 9$ $\quad$ When $x = 9$, $23 - x = 14$

Hence required numbers are 14, 9 or 9, 14

-:7.5:-

Sum of two numbers is 12 and the sum of their squares is 74. Find the numbers.

SOLUTION

Let The first number = x, then
The second number = 12 – x

$$x^2 + (12 - x)^2 = 74$$
$$x^2 + 144 - 24x + x^2 = 74$$
$$2x^2 - 24x + 144 = 74$$
$$2x^2 - 24x + 144 - 74 = 0$$
$$2x^2 - 24x + 70 = 0$$
$$x^2 - 12x + 35 = 0$$
$$x^2 - 7x - 5x + 35 = 0$$
$$x(x - 7) - 5(x - 7) = 0$$
$$(x - 7)(x - 5) = 0$$

$x - 7 = 0$ or $x - 5 = 0$

$x = 7$ $\quad$ $x = 5$

Hence required numbers are 7, 5 or 5, 7

-:7.6:-

Find two numbers such that their sum is 10 and their product is 22.

SOLUTION

Let The first number = x, then

The Second number = 10 − x

$$x(10-x)=22$$

$$10x-x^2=22$$

$$10x-x^2-22=0$$

$$-x^2+10x-22=0$$

$$x^2-10x+22=0$$

Using quadratic formula, we get

a = 1, b = -10 and c = 22

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(22)}}{2(1)}$$

$$=\frac{10\pm\sqrt{100-88}}{2}=\frac{10\pm\sqrt{12}}{2}=\frac{10\pm2\sqrt{3}}{2}$$

$$=5\pm\sqrt{3}$$

Hence the required numbers are $5+\sqrt{3}, 5-\sqrt{3}$

-:7.7:-

Two numbers differ by 7 and have a product of 120. What are they?

SOLUTION

Let The smaller number be x, then the larger number will be x + 7

$$x(x+7)=120$$

$$x^2+7x=120$$

$$x^2+7x-120=0$$

$$x^2+15x-8x-120=0$$

$$x(x+15)-8(x+15)=0$$

$$(x+15)(x-8)=0$$

$x + 15 = 0$ or $x - 8 = 0$

$x = -15$ $x = 8$

The numbers are either 8 and 8 + 7 = 15 or the numbers are 15 and -15 + 7 = -8

-:7.8:-

Find three consecutive integers the sum of whose squares is 509.

SOLUTION

Let the three consecutive number be x, (x+1), (x+2)

Then

$$x^2 + (x+1)^2 + (x+2)^2 = 509$$

$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 509$$

$$3x^2 + 6x + 5 = 509$$

$$3x^2 + 6x + 5 - 509 = 0$$

$$3x^2 + 6x - 504 = 0$$

$$x^2 + 2x - 168 = 0$$

$$x^2 + 2x - 168 = 0$$

Here a = 1, b = 2 and c = -168

$$x^2 + 14x - 12x - 168 = 0$$

$$x(x+14) - 12(x+14) = 0$$

$$(x+14)(x-12) = 0$$

$x + 14 = 0$ or $x - 12 = 0$

$x = -14$ $x = 12$

The consecutive numbers are 12, 13, 14 or -12, -13, -14
Whose sum of squares is 509.

-:7.9:-

Find the number which added to 22 and 40 gives two numbers with a product 1288.

SOLUTION

Let the number be x, then

$$(x+22) + (x+40) = 1288$$

$$x^2 + 40x + 22x + 880 = 1288$$

$$x^2 + 62x + 880 - 1288 = 0$$

$$x^2 + 62x - 408 = 0$$

$$x^2 + 68x - 6x - 408 = 0$$

$$x(x + 68) - 6(x + 68) = 0$$

$$(x + 68)(x - 6) = 0$$

$$x + 68 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = -68 \qquad\qquad x = 6$$

The numbers is either 6 or -68 as (6+22)(6+40)=28×46=1288 and (-68+22)(-68+40) = (-46)(-28) = 1288.

-:7.10:-

Two positive integers differ by 6, and their product is 667. Find the numbers.

SOLUTION

Let The smaller number be x, then the larger number will be x + 6

$$x(x + 6) = 667$$

$$x^2 + 6x - 667 = 0$$

$$x^2 + 29x - 23x - 667 = 0$$

$$x(x + 29) - 23(x + 29) = 0$$

$$(x + 29)(x - 23) = 0$$

$$x + 29 = 0 \quad \text{or} \quad x - 23 = 0$$

$$x = -29 \qquad\qquad x = 23$$

Hence required numbers are $x = 23$ and $x + 6 = 23 + 6 = 29$

-:7.11:-

A garden contains 90 trees. The number of trees in each row is 3 more than twice the number of rows. Find the number of rows and number of trees per row.

SOLUTION

Let x represent the number of rows. Then 2x + 3 represent the number of trees per row.

Total number of trees are equal to the number of rows multiplied by the number of trees per row. So

$$x(2x + 3) = 90$$

$$2x^2 + 3x = 90$$

$$2x^2 + 3x - 90 = 0$$
$$2x^2 + 15x - 12x - 90 = 0$$
$$x(2x + 15) - 6(2x + 15) = 0$$
$$(2x + 15)(x - 6) = 0$$

$2x + 15 = 0$ or $x - 6 = 0$

$x = -\dfrac{15}{2}$ $\quad x = 6$

The solution -15/2 must be disregarded, so there are 6 rows and $2x + 3$ or $2(6) + 3 = 15$ trees per row.

-:7.12:-

A page for a Sunday Magazine contains 70 square inches of type. The height of a page is twice the width. If the margin around the type is to be 2 inches uniformly, what are the dimensions of a page?

SOLUTION

Let x represent the width of a page. Then 2x represent the height of a page. So

Width of the typed material = x – 4 and height of the typed material = 2x – 4

Width of typed material x Height of typed material = Area of typed material

$$(x - 4)(2x - 4) = 70$$
$$2x^2 - 4x - 8x + 16 = 0$$
$$2x^2 - 12x + 16 - 70 = 0$$
$$2x^2 - 12x - 54 = 0$$
$$x^2 - 6x - 27 = 0$$
$$x^2 - 9x + 3x - 27 = 0$$
$$x(x - 9) + 3(x - 9) = 0$$
$$(x - 9)(x + 3) = 0$$

$x - 9 = 0$ or $x + 3 = 0$

$x = 9$ $\quad x = -3$

The negative solution has to be disregarded; thus the page is 9 inches wide and its height is 2(9) = 18 inches.

-:7.13:-

The perimeter of a rectangle is 44 feet and its area is 112 square feet. Find the length and width of the rectangle.

SOLUTION

Let the width be W feet. Then the length is 22 – W feet. Area of rectangle is

$$W(22 - W) = 112$$
$$22W - W^2 = 112$$
$$22W - W^2 - 112 = 0$$
$$-W^2 + 22W - 112 = 0$$
$$W^2 - 22W + 112 = 0$$
$$W^2 - 14W - 8W + 112 = 0$$
$$W(W - 14) - 8(W - 14) = 0$$
$$(W - 14)(W - 8) = 0$$

$W - 14 = 0$ or $W - 8 = 0$

$W = 14$ $W = 8$

If width is 8 feet then the length is $22 - 8 = 14$ feet

-:7.14:-

Find the dimensions of a rectangular field which has an area of 240 m^2 and a perimeter of 64 m.

SOLUTION

Let the width be W meters, then the length is (32-W) meters, Hence

$$W(32 - W) = 240m^2$$
$$32W - W^2 = 240$$
$$W^2 - 32W + 240 = 0$$
$$W^2 - 12W - 20W + 240 = 0$$
$$W(W - 12) - 20(W - 12) = 0$$
$$(W - 12)(W - 20) = 0$$

$W - 12 = 0$ or $W - 20 = 0$

$W = 12$ $W = 20$

If width is 12 m then the length is 20 m.

-:7.15:-

A woman walks 20 km at a certain speed and then 10km at 3km/h faster. If the total time taken is 12 hrs. find her original speed.

SOLUTION

Let The original speed by x km/h

Time for 20 km = $\dfrac{\text{Distance}}{\text{Speed}} = \dfrac{20}{x}$ hrs

Time for 10 km = $\dfrac{\text{Distance}}{\text{Speed}} = \dfrac{10}{x+3}$ hrs

Total time is as = $\dfrac{10}{x+3} + \dfrac{20}{x} = 12$

Multiplying both sides by x(x+3) and simplifying, we get

$$10x + 20(x+3) = 12x(x+3)$$

$$10x + 20x + 60 = 12x^2 + 36x$$

$$30x + 60 = 12x^2 + 36x$$

$$-12x^2 + 30x - 36x + 60 = 0$$

$$-12x^2 - 6x + 60 = 0$$

$$12x^2 + 6x - 60 = 0$$

$$2x^2 + x - 10 = 0$$

$$2x^2 + 5x - 4x - 10 = 0$$

$$x(2x+5) - 2(2x+5) = 0$$

$$(2x+5)(x-2) = 0$$

$2x + 5 = 0$ or $x - 2 = 0$

$x = -\dfrac{5}{2}$ $x = 2$

Hence original speed was 2 km/h

EXERCISE NO. 8

SET – A

-:8.1:-

Solve for x and y; $x + 4y = 14, 3x - 2y = 7$

SOLUTION

$$x + 4y = 14 \quad ....(I)$$
$$3x - 2y = 7 \quad ....(II)$$

Eliminate y multiplying eq (II) by 2 and adding

$$\begin{array}{rcl} x + 4y &=& 14 \\ 6x - 4y &=& 14 \\ \hline 7x &=& 28 \\ x &=& 4 \end{array}$$

Substituting $x = 4$ in eq (I) we get

$$4 + 4y = 14$$
$$4y = 14 - 4$$
$$4y = 10$$
$$y = \frac{10}{4} = \frac{5}{2}$$

Hence $x = 4, y = {}^{5}/_{2}$

-:8.2-

Solve for x and y; $3x + 2y = 18, 5x + 7y = 41$

SOLUTION

$$3x + 2y = 18 \quad ....(1)$$
$$5x + 7y = 41 \quad ....(2)$$

To eliminate x, multiplying eq.(1) by 5 and eq(2) by 3 and subtracting

$$\begin{array}{rcl} 15x + 10y &=& 90 \\ -15x \pm 21y &=& -123 \\ \hline -11y &=& -33 \\ y &=& 3 \end{array}$$

Substituting $y = 3$ in eq (1)

$$3x + 2(3) = 18$$
$$3x + 6 = 18$$
$$3x = 18 - 6 = 12$$
$$x = 4$$

Hence x = 4, y = 3

-:8.3:-

Solve for x and y; 2x + 5y = 30, 3x - 2y = 7

SOLUTION

$$2x + 5y = 30 \quad \ldots\ldots\ldots(1)$$
$$3x - 2y = 7 \quad \ldots\ldots\ldots(2)$$

To eliminate y, multiplying eq (1) by 2 and eq (2) by 5 and adding

$$4x + 10y = 60$$
$$15x - 10y = 35$$
$$19x = 95$$
$$x = 5$$

Substituting x = 5 in eq (1), we get

$$2(5) + 5y = 30$$
$$10 + 5y = 30$$
$$5y = 30 - 10$$
$$5y = 20$$
$$y = 4$$

Hence x = 5, y = 4

-:8.4:-

Solve for x and y; 3x + 2y = 106, 2x + 4y = 92

SOLUTION

$$3x + 2y = 106 \quad \ldots\ldots\ldots(1)$$
$$2x + 4y = 92 \quad \ldots\ldots\ldots(2)$$

To eliminate x, multiplying eq (1) by 2 and eq (2) by 3 and subtracting.

$$6x + 4y = 212$$
$$\underline{-6x \pm 12y = -276}$$
$$-8y = -64$$
$$y = 8$$

Substituting y = 8 in eq (2)

$$2x + 4(8) = 92$$
$$2x + 32 = 92$$
$$2x = 92 - 32 = 60$$
$$x = 30$$

Hence x = 30, y = 8

-:8.5:-

Solve for x and y; $x + 2y = 11$, $2x + 3y = 19$

SOLUTION

$$x + 2y = 11 \quad \text{...........(1)}$$
$$2x + 3y = 19 \quad \text{...........(2)}$$

To eliminate x, multiplying eq (1) by 2 and subtracting eq (2) from it.

$$\begin{array}{r} 2x + 4y = 22 \\ -2x \pm 3y = -19 \\ \hline y = 3 \end{array}$$

Substituting y =3 in eq (1), we get

$$x - 2(3) = 11$$
$$x + 6 = 11$$
$$x = 11 - 6$$
$$x = 5$$

Hence $x = 5$, $y = 3$

-:8.6:-

Solve for x and y; $2x - 3y = -11$, $3x + 5y = 31$

SOLUTION

$$2x - 3y = -11 \quad \text{...........(1)}$$
$$3x + 5y = 31 \quad \text{...........(2)}$$

To eliminate y, multiplying eq (1) by 5 and eq 2 by 3 and adding

$$\begin{array}{r} 10x - 15y = -55 \\ 9x + 15y = 93 \\ \hline 19x = 38 \end{array}$$
$$x = 2$$

Substituting $x = 2$ in eq (1), we get

$$2(2) - 3y = -11$$
$$4 - 3y = -11$$
$$-3y = -11 - 4$$
$$-3y = -15$$
$$y = 5$$

Hence $x = 2$, $y = 5$

-:8.7:-

Solve for x and y; $3x - 2y = 5$, $4x + 3y = 18$

SOLUTION

$$3x - 2y = 5 \quad \ldots\ldots\ldots\ldots(1)$$
$$4x + 3y = 18 \quad \ldots\ldots\ldots\ldots(2)$$

To eliminate y, multiplying eq (1) by 3 and eq (2) by 2 and adding.

$$9x - 6y = 15$$
$$\underline{8x + 6y = 36}$$
$$17x = 51$$
$$x = 3$$

Substituting x = 3 in eq (2)

$$4(3) + 3y = 18$$
$$12 + 3y = 18$$
$$3y = 18 - 12$$
$$3y = 6 \Rightarrow y = 2$$

Hence x =3, y = 2

-:8.8:-

Solve for x and y; 4x + 7y = 11, x - 2y = -16

SOLUTION

$$4x + 7y = 11 \quad \ldots\ldots\ldots\ldots(1)$$
$$x - 2y = -18 \quad \ldots\ldots\ldots\ldots(2)$$

To eliminate x, multiplying eq (2) by 4 and subtracting from eq (1)

$$4x + 7y = 11$$
$$-\ \underline{4x \overset{+}{-} 8y = \overset{+}{-}64}$$
$$15y = 75$$
$$y = 5$$

Substituting y = 5 in eq (2)

$$x - 2(5) = -16$$
$$x - 10 = -16$$
$$x = -16 + 10$$
$$x = -6$$

Hence x= -6, y = 5

-:8.9:-

Solve for x and y; 3x + 5y = 31, 2x + 3y = 19

SOLUTION

$$3x + 5y = 31 \quad \ldots\ldots\ldots\ldots(1)$$
$$2x + 3y = 19 \quad \ldots\ldots\ldots\ldots(2)$$

Multiplying eq(1) by 2 and eq (2) by 3 and subtracting, we get

$$\begin{array}{r} 6x + 10y = 62 \\ -\;\; 6x \; \underset{-}{+} \; 9y = \underset{-}{57} \\ \hline y = 5 \end{array}$$

For x, putting y = 5 in eq (1), we get

$$3x + 5y = 31$$
$$3x + 25 = 31$$
$$3x = 31 - 25$$
$$3x = 6$$
$$x = 2$$

Hence x = 2, y = 5

-:8.10:-

Solve for x and y; 2x - y = -5, x + 3y = 29

SOLUTION

$$2x - y = -5 \;...........(1)$$
$$x + 3y = 29 \;...........(2)$$

To eliminate y, multiplying eq (1) by 3 and adding

$$\begin{array}{r} 6x - 3y = -15 \\ x + 3y = 29 \\ \hline 7x = 14 \\ x = 2 \end{array}$$

Substituting x = 2 in eq (2)

$$2 + 3y = 29$$
$$3y = 29 - 2$$
$$3y = 27$$
$$y = 9$$

Hence x = 2, y = 9

-:8.11:-

Solve for x and y; 5x + 7y = 11, 2x - 4y = -16

SOLUTION

$$5x + 7y = 11 \;............(1)$$
$$2x - 4y = -16 \;............(2)$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 7 and adding

$$20x + 28y = 44$$
$$14x - 28y = -112$$
$$34x = -68$$
$$x = -2$$

Substituting $x = -2$ in eq (1), we get

$$5(-2) + 7y = 11$$
$$-10 + 7y = 11$$
$$7y = 11 + 10$$
$$7y = 21$$
$$y = 3$$

Hence $x = -2, y = 3$

-:8.12:-

Solve for x and y; $x + y = 6$, $x + y = 9$

SOLUTION

$$x + y = 6$$
$$x + y = 9$$

There is no common solution for the equations. The equations are inconsistent

-:8.13:-

Solve for x and y; $2(x-4) = 3(y-3)$, $y - 2x = -13$

SOLUTION

$$2(x - 4) = 3(y - 3) \text{ ....(1)}$$
$$y - 2x = -13 \text{ ....(2)}$$

First, we simplify the eq (1) as below.

$$2x - 8 = 3y - 9$$
$$2x - 3y = -9 + 8$$
$$2x - 3y = -1$$

Now the equations are

$$2x - 3y = -1$$
$$-2x + y = -13$$

To eliminate x, adding eq (1) and eq (2)

$$-2y = -14$$
$$y = 7$$

Substitute $y = 7$ in eq (2), we get

$$-2x + 7 = -13$$
$$-2x = -13 - 7$$
$$-2x = -20$$
$$x = 10$$

Hence $x = 10, y = 7$

-:8.14:-

Solve for x and y; $3x - 2(y+3) = 2$, $2(x-3) + 4 = 3y - 5$

SOLUTION

$$3x - 2(y + 3) = 2$$
$$2(x - 3) + 4 = 3y - 5$$

First we set the equations as

$$3x - 2y - 6 = 2$$
$$3x - 2y = 2 + 6$$
$$3x - 2y = 8$$

and

$$2x - 6 + 4 = 3y - 5$$
$$2x - 3y = -5 + 6 - 4$$
$$2x - 3y = -3$$

Hence equations are

$$3x - 2y = 8 \text{ ....(1)}$$
$$2x - 3y = -3 \text{ ....(2)}$$

To eliminate x, multiply eq(1) by 2 and eq (2) by 3 and subtracting.

$$6x - 4y = 16$$
$$-\;\underline{6x - 9y = -9}$$
$$5y = 25$$
$$y = 5$$

Substituting $y = 5$ in eq (1)

$$3x - 2(5) = 8$$
$$3x - 10 = 8$$
$$3x = 8 + 10$$
$$3x = 18$$
$$x = 6$$

Hence $x = 6, y = 5$

-:8.15:-

Solve for x and y; $2x - 3y = 7$, $9x + 3y = 15$

SOLUTION

$$2x - 3y = 7 \quad \ldots\ldots\ldots\ldots(1)$$
$$9x + 3y = 15 \quad \ldots\ldots\ldots\ldots(2)$$

To eliminate y, adding eq (1) and eq (2)

$$11x = 22$$
$$x = 2$$

Substituting $x = 2$ in eq (1)

$$2(2) - 3y = 7$$
$$4 - 3y = 7$$
$$-3y = 7 - 4$$
$$-3y = 3$$
$$y = -1$$

Hence $x = 2$, $y = -1$

-:8.16:-

Solve for x and y; $4x + 2y = 5$, $5x - 3y = -2$

SOLUTION

$$4x + 2y = 5 \quad \ldots\ldots\ldots\ldots(1)$$
$$5x - 3y = -2 \quad \ldots\ldots\ldots\ldots(2)$$

To eliminate y, multiplying eq (1) by 3 and eq (2) by 2 and adding.

$$12x + 6y = 15$$
$$\underline{10x - 6y = -4}$$
$$22x = 11$$
$$x = \frac{1}{2}$$

Substituting $x = 1/2$ in eq (1)

$$4(\tfrac{1}{2}) + 2y = 5$$
$$2 + 2y = 5$$
$$2y = 5 - 2$$
$$2y = 3 \Rightarrow y = \frac{3}{2}$$

Hence $x = 1/2$, $y = 3/2$

-:8.17:-

Solve for x and y; $4x - 3y = 1, 3x + 4y = -18$

SOLUTION

$$4x - 3y = 1 \quad \text{............}(1)$$
$$3x + 4y = -18 \quad \text{............}(2)$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 3 and adding.

$$16x - 12y = 4$$
$$9x + 12y = -54$$
$$25x = -50$$
$$x = -2$$

Substituting $x = -2$ in eq (2).

$$3(-2) + 4y = -18$$
$$-6 + 4y = -18$$
$$4y = -18 + 6$$
$$4y = -12$$
$$y = -3$$

Hence $x = -2, y = -3$

-:8.18:-

Solve for x and y; $2x + y = 1, y = 5 - x$

SOLUTION

$$2x + y = 1$$
$$y = 5 - x$$

OR

$$2x + y = 1$$
$$x + y = 5$$

To eliminate y, subtracting the eq (2) from eq (1)

$$2x + y = 1$$
$$x + y = 5$$
$$x = -4$$

Substituting $x = -4$ in eq (2)

$$y = 5 - (-4)$$
$$y = 5 + 4$$
$$y = 9$$

Hence $x = -4, y = 9$

-:8.19:-

Solve for x and y; 7x – 6y = 20, 3x – 2y -8 0

SOLUTION

$$7x - 6y = 20 \quad ....(1)$$
$$3x - 2y = 8 \quad ....(2)$$

To eliminate y, multiplying eq (1) by 2 and eq(2) by 6 and subtracting

$$14x - 12y = 40$$
$$-\ \underline{18x \overset{+}{-} 12y = \overset{-}{}48}$$
$$-4x = -8$$
$$x = 2$$

For y, putting x = 2 in eq (1), we get

$$7(2) + 6y = 20$$
$$14 - 6y = 20$$
$$-6y = 20 - 14$$
$$-6y = 6$$
$$y = -1$$

Hence $x = 2, y = -1$

-:8.20:-

Solve for x and y; 3x + 2y = 44, 2x + 4y = 56

SOLUTION

$$3x + 2y = 44 \quad ....(1)$$
$$2x + 4y = 56 \quad ....(2)$$

Multiplying equation (1) by 2, so that the co-efficient of y in first equation is the same as that of in the second equation i.e. by subtracting

$$6x + 4y = 88$$
$$-\ \underline{2x \pm 4y = \overset{-}{}56}$$
$$4x = 32$$
$$x = 8$$

for y, substitute x = 8 in equation (1) or (2), we get

$$3(8)+2y=44$$
$$2y=44-24$$
$$2y=20 \Rightarrow y=10$$

Hence the required values are x = 8 and y = 10

-:8.21:-

Solve for x and y; $\frac{x}{2}+\frac{2y}{3}=-4,\ \frac{x}{4}-\frac{3y}{2}=20$

SOLUTION

$$\frac{x}{2}+\frac{2y}{3}=-4 \quad ....(1)$$
$$\frac{x}{4}-\frac{3y}{2}=20 \quad ....(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) both side by 6 and equation (2) both side by 4, we get

$$\frac{x}{2}+\frac{2y}{3}=-4$$

Multiplying both sides by 6

$$3x+4y=-24$$
$$\frac{x}{4}-\frac{3y}{2}=20$$

Multiplying both sides by 4

$$x-6y=80$$

Now we can proceed as follows:

$$3x+4y=-24 \ ....(3)$$
$$x-6y=80 \quad ....(4)$$

Multiplying equation (4) by –3 and adding equation (3) to it, we get

$$3x+4y=-24$$
$$\underline{-3x+18y=-240}$$
$$22y=-264$$
$$y=-12$$

For x, substitute y = –12 in equation (4), we get

$$x - 6(-12) = 80$$
$$x + 72 = 80$$
$$x = 80 - 72 \Rightarrow x = 8$$

Hence the required set is $x = 8$ and $y = -12$

Check:

$$\frac{x}{2} + \frac{2y}{3} = -4 \quad \text{and} \quad \frac{x}{4} - \frac{3y}{2} = 20$$

$$\frac{8}{2} + \frac{2(-12)}{3} = -4 \qquad \frac{8}{2} + \frac{3(-12)}{2} = 20$$

$$4 - 8 = -4 \qquad 2 + 18 = 20$$

$$-4 = -4 \qquad 20 = 20$$

-:8.22:-

Solve for x and y; $\frac{x}{4} - \frac{2y}{3} = -3,\ \frac{x}{3} + \frac{y}{3} = 7$

SOLUTION

$$\frac{x}{4} - \frac{2y}{3} = -3 \text{..............} (1)$$

$$\frac{x}{3} + \frac{y}{3} = 7 \text{..............} (2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 12 and equation (2) by 3.

$$3x - 8y = -36 \text{..............} (3)$$

$$x + y = 21 \text{..............} (4)$$

Again multiplying equation (4) by 8 and adding, we get

$$3x - 8y = -36$$
$$8x + 8y = 168$$
$$11x = 132$$
$$x = 12$$

For y, putting $x = 12$ in equation (4), we get

$$12 + y = 21$$
$$y = 21 - 12 = 9$$

Hence the required solution is $x = 12$ and $y = 9$

-:8.23:-

Solve for x and y; $\frac{x}{2}-\frac{y}{3}=\frac{1}{4}, \ \frac{x}{4}+\frac{y}{5}=\frac{1}{2}$

SOLUTION

$$\frac{x}{2}-\frac{y}{3}=\frac{1}{4} \quad ....(1)$$

$$\frac{x}{4}+\frac{y}{5}=\frac{1}{2} \quad ....(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 12 and equation (2) by 20, we get

$$6x-4y=3 \quad ....(3)$$

$$5x+4y=10 \quad ....(4)$$

By adding equation (3) and equation (4), we get

$$\begin{array}{r} 6x-4y=13 \\ 5x+4y=10 \\ \hline 11x=13 \end{array}$$

$$x=\frac{13}{11}$$

For y, putting x = 13/11 in equation (4), we get

$$5\left(\frac{13}{11}\right)+4y=10$$

$$\frac{65}{11}+4y=10$$

$$4y=10-\frac{65}{11}=\frac{110-65}{11}$$

$$4y=\frac{45}{11} \Rightarrow y=\frac{45}{44}$$

Hence the required solution is x = 13/11, and y = 45/44

-:8.24:-

Solve for x and y; $\frac{3x}{2}-\frac{2y}{7}=-1,\ 4x+y=2$

SOLUTION

$$\frac{3x}{2}-\frac{2y}{7}=-1 \quad ....(1)$$

$$4x+y=2 \quad ....(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 14 and equation (2) by 4 and adding

$$21x-4y=14$$
$$16x+4y=8$$
$$\overline{37x=-6}$$

$$x=-\frac{6}{37}$$

For y, putting x = -6/37 in equation (4), we get

$$4\left(\frac{-6}{37}\right)+y=2$$

$$-\frac{24}{37}+y=2$$

$$y=2+\frac{24}{37}=\frac{74+24}{37}=\frac{98}{37}$$

Hence the required solution is x = -6/37, and y = 98/37

-:8.25:-

Solve for x and y; $\frac{4x}{5}-\frac{3y}{2}=\frac{1}{5},\ -2x+y=-1$

SOLUTION

$$\frac{4x}{5}-\frac{3y}{2}=\frac{1}{5} \quad ....(1)$$

$$-2x+y=-1 \quad ....(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 10 and equation (2) by 15 and adding

$$\begin{aligned} 8x - 15y &= 2 \\ -30x + 15y &= -15 \\ \hline -22x &= -13 \\ x &= \frac{13}{22} \end{aligned}$$

For y, putting x = 13/22 in equation (2), we get

$$-2\left(\frac{13}{22}\right) + y = 1$$

$$-\frac{13}{11} + y = -1$$

$$y = -1 + \frac{13}{11} = \frac{-11+13}{11} = \frac{2}{11}$$

Hence the required solution is x = 13/22, and y = 2/11

-:8.26:-

Solve for x and y;

SOLUTION

$$\frac{3}{x} + \frac{2}{y} = 2 \quad ....(1)$$

$$\frac{2}{x} - \frac{3}{y} = \frac{1}{4} \quad ....(2)$$

It is not a system of linear equations but can be transferred into a linear system by changing variables. Let we substitute $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the above system becomes:

$$3u + 2v = 2 \quad ....(3)$$

$$2u - 3v = \frac{1}{4} \quad ....(4)$$

The new system can be solved simultaneously as:

Multiplying equation (3) by 3 and equation (4) by 2, adding them, we get

$$9u + 6v = 6$$
$$4u - 6v = \frac{1}{2}$$
$$13u = 6 + \frac{1}{2} = \frac{13}{2}$$
$$u = \frac{1}{2}$$

For v, putting x = 1/2 in equation (3), we get

$$3\left(\frac{1}{2}\right) + 2v = 2 \Rightarrow \frac{3}{2} + 2v = 2$$
$$2v = 2 - \frac{3}{2} = \frac{11}{2} \Rightarrow v = \frac{1}{4}$$

Since $u = \frac{1}{x}$ and $v = \frac{1}{y}$, we have

$$\frac{1}{x} = \frac{1}{2} \Rightarrow x = 2 \text{ and } \frac{1}{y} = \frac{1}{4} \Rightarrow y = 4$$

-:8.27:-

Solve for x and y; $\frac{2}{x} - \frac{7}{y} = \frac{9}{10}, \frac{5}{x} + \frac{4}{y} = -\frac{41}{20}$

SOLUTION

$$\frac{2}{x} - \frac{7}{y} = \frac{9}{10} \quad \text{...............(1)}$$
$$\frac{5}{x} + \frac{4}{y} = -\frac{41}{20} \quad \text{...............(2)}$$

It is not a system of linear equations but can be transferred into a linear system by changing variables. Let we substitute $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the above system becomes:

$$2u - 7v = \frac{9}{10} \quad \text{...............(3)}$$
$$5u + 4v = -\frac{41}{20} \quad \text{...............(4)}$$

The new system can be solved simultaneously as:

Multiplying equation (3) by 4 and equation (4) by 7, adding them, we get

$$8u - 28v = \frac{18}{5}$$

$$35u + 28v = -\frac{287}{20}$$

$$43u = \frac{18}{5} - \frac{287}{20} = \frac{72-287}{20}$$

$$43u = -\frac{215}{20} = -\frac{43}{4}$$

$$u = -\frac{1}{4}$$

For v, putting x = -1/4 in equation (3), we get

$$2\left(-\frac{1}{4}\right) - 7v = \frac{9}{10} \Rightarrow -\frac{1}{2} - 7v = \frac{9}{10}$$

$$-7v = \frac{9}{10} + \frac{1}{2} = \frac{9+5}{10}$$

$$-7v = \frac{14}{10} \Rightarrow v = \frac{2}{10} = -\frac{1}{5}$$

Since $u = \frac{1}{x}$ and $v = \frac{1}{y}$, we have

$$\frac{1}{x} = -\frac{1}{4} \Rightarrow x = -4 \text{ and } \frac{1}{y} = -\frac{1}{5} \Rightarrow y = -5$$

Hence solution set is x = -4 and y = -5

SET – B

-:8.1:-

The price of 2 balls and 3 bats is Rs. 200 and the price of 5 balls and 4 bats is Rs. 290. Find the price of the ball and the price of the bat respectively.

SOLUTION

Let the price of a ball = Rs. x and
The price of a bat = Rs. y, then

$$2x + 3y = 200 \quad \text{..........(1)}$$
$$5x + 4y = 290 \quad \text{..........(2)}$$

Multiplying equation (1) by 4 and equation (2) by 3 and subtracting it, we get

$$8x + 12y = 800$$
$$\underline{\mathop{-}15x \mathop{+}\limits_{-} 12y = \mathop{870}\limits_{-}}$$
$$-7x = -70$$
$$x = 10$$

For y, putting x = 10 in eq (1), we get

$$2(10) + 3y = 200$$
$$20 + 3y = 200$$
$$3y = 200 - 20$$
$$3y = 180$$
$$y = 60$$

Hence The price of ball = Rs. 10 and
The price of bat = Rs. 60

-:8.2:-

The difference of two numbers is 4. Twice the first number plus three times the second equals 28. Find the two number.

SOLUTION

Let the larger number = x and
The smaller number = y, then

$$x - y = 4 \quad \text{..........(1)}$$
$$2x + 3y = 28 \quad \text{..........(2)}$$

Multiplying equation (1) by 3 and adding equation (2) to it. we get

$$3x - 3y = 12$$
$$\underline{2x + 3y = 28}$$
$$5x = 40$$
$$x = 8$$

For y, putting $x = 8$ in eq (1). we get

$$8 - y = 4$$
$$-y = 4 - 8$$
$$y = -4$$
$$y = 4$$

Hence The larger number $= x = 8$ and

The smaller number $= y = 4$

-:8.3:-

There are two numbers such that the sum of the first and three times the second is 53, while the difference between 4 times the first and twice the second is 2. Find the numbers.

SOLUTION

Let the numbers are x and y

$$x + 3y = 53 \text{ ....(1)}$$
$$4x - 2y = 2 \text{ ....(2)}$$

To eliminate x, multiplying eq (1) by 4 and subtracting eq (2) from it

$$4x + 12y = 212$$
$$\underline{4x - 2y = 2}$$
$$14y = 210$$
$$y = 15$$

Substituting $y = 15$ in eq (1).

$$x + 3(15) = 53$$
$$x + 45 = 53$$
$$x = 53 - 45$$
$$x = 8$$

Hence the two numbers are $x = 8, y = 15$

-:8.4:-

If the numerator of a certain fraction is increased by 5 and the denominator is decreased by 1, the resulting fraction is 8/3. However, if the numerator of the original fraction is doubled and the denominator is increased by 7, the resulting fraction is 6/11. Find the original fraction.

SOLUTION

Let the numerator = x and
The denominator = y, then

The fraction = $\frac{x}{y}$

According to conditions, we have the following fractions:

$$\frac{x+5}{y-1} = \frac{8}{3} \quad \text{..............(A)}$$

$$\frac{2x}{y+7} = \frac{6}{11} \quad \text{.............(B)}$$

From (A), by cross multiplication

$$3(x+5) = 8(y-1)$$
$$3x + 15 = 8y - 8$$
$$3x - 8y = -8 - 15$$
$$3x - 8y = -23$$

From fraction (B) by cross multiplication

$$22x = 6y + 42$$
$$22x - 6y = 42$$
$$11x - 3y = 21$$

Now

$$3x - 8y = -23 \quad \text{..............(1)}$$
$$10x - 3y = 21 \quad \text{................(2)}$$

To eliminate x multiplying equation (1) by 11, and multiplying equation (2) by 3, subtracting it, we get

$$33x - 88y = -253$$
$$\underline{{}_{-}33x \overset{+}{-} 9y = \underset{-}{63}}$$
$$-79y = -316$$
$$y = 4$$

For x, putting y = 4 in equation (1)

$$3x - 32 = -23$$
$$3x = -23 + 32$$
$$3x = 9$$
$$x = 3$$

Hence fraction is 3/4

-:8.5:-

Sadaf bought 3 packages of slanti and 4 bags of potato chips for Rs. 125. Later she bought 2 more package of slanti and 5 bags of potato chips for Rs. 130, Find the price of a package of slanti and one bag of potato chips.

SOLUTION

Let the price of package of Slanti = Rs. x and
The price of bag of potato chips = Rs. y, then

$$3x + 4y = 125 \quad \text{...........(1)}$$
$$2x + 5y = 130 \quad \text{...........(2)}$$

To eliminate x, multiplying equation (1) by 2 and equation (2) by 3, subtracting, we get

$$6x + 8y = 250$$
$$\underline{6x + 15y = 390}$$
$$-7y = -140$$
$$x = 20$$

For x, putting y = 20 in eq (1), we get

$$3x + 4(20) = 125$$
$$3x + 80 = 125$$
$$3x = 125 - 80$$
$$3x = 45$$
$$x = 15$$

Hence The price of Package of Slanti = x = Rs. 15 and
The price of Bag of Chips = y = Rs. 20

-:8.6:-

Find the cost of a ruler and a pen if 3 rulers and 10 pens cost Rs. 260, and 2 rules and five pens cost Rs. 135.

SOLUTION

Let the cost of a Ruler = Rs. x and
The cost of a Pen = Rs. y, then

$$3x + 10y = 260 \quad ....(1)$$
$$2x + 5y = 135 \quad ....(2)$$

To eliminate x, multiplying equation (1) by 2 and subtracting, we get

$$\begin{array}{r} 3x + 10y = 260 \\ \underline{-4x \pm 10y = -270} \\ -x = -10 \\ x = 10 \end{array}$$

For y, putting x = 10 in eq (1), we get

$$3(10) + 10y = 260$$
$$30 + 10y = 260$$
$$10y = 260 - 30$$
$$10y = 230$$
$$y = 23$$

Hence The price of Ruler = x = Rs. 10 and
The price of Pen = y = Rs. 23

-:8.7:-

The cost of tables and 8 chairs is Rs. 4350 and the cost of 2 tables and 5 chairs is Rs. 2800. Find the cost of one table and one chair.

SOLUTION

Let the price of Table = Rs. x and
The price of Chair = Rs. y, then

$$3x + 8y = 4350 \quad ....(1)$$
$$2x + 5y = 2800 \quad ....(2)$$

To eliminate x, multiplying equation (1) by 2 and equation (2) by 3, subtracting, we get

$$\begin{array}{r} 6x + 16y = 8700 \\ \underline{-6x \pm 15y = -8400} \\ y = 300 \end{array}$$

For x. putting $y = 300$ in eq (1), we get

$$3x + 8(300) = 4350$$
$$3x + 2400 = 14350$$
$$3x = 4350 - 2400$$
$$3x = 1950$$
$$x = 650$$

Hence The price of a Table = x = Rs. 650

The price of a Chair = y = Rs. 300

-:8.8:-

The boys in a certain district collected Rs. 30720 from their annual blanket sale. If there were 80 boys in all, each boy scout collected Rs. 240, and each campfire boy collected Rs. 480, how many boys in each organization sold blankets?

SOLUTION

Let the number of Boy Scout = x and

The number of Campfire Boy =. Y

So

$$x + y = 80$$

Amount of Boy Scouts + Amount of Campfire Boy = Total Amount

$$240x + 480y = 30720$$

Hence we have two equations as follows:

$$x + y = 80 \quad ....(1)$$
$$240x + 480y = 30720 \quad ....(2)$$

To eliminate y, multiplying equation (1) by 480 and, subtracting, we get

$$480x + 480y = 38400$$
$$\underline{\underset{-}{}\,240x \underset{-}{+} 480y = \underset{-}{30720}}$$
$$240y = 7680$$
$$x = 32$$

For y, putting $x = 32$ in eq (1), we get

$$32 + y = 80$$
$$y = 80 - 32$$
$$y = 48$$

Hence The number of Boy Scout = x = 32 and
The number of campfire Boy = y = 48

-:8.9:-

Six bags of cement and two bags of sand weight 23 kg and four bags of cement and three bags of sand weight 17 kg. Find the weight of a bag of cement and a bag of sand.

SOLUTION

Let x = The weight of cement bag
y = The weight of sand bag

Thus

$$6x + 2y = 23 \text{ .... (1)}$$
$$4x + 3y = 17 \text{ .... (2)}$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 6 and subtracting

$$\begin{array}{r} 24x + 8y = 92 \\ -24x \pm 18y = -102 \\ \hline -10y = -10 \\ y = 1 \end{array}$$

For x, putting y = 1 in eq (1), we get

$$6x + 2(1) = 23$$
$$6x + 2 = 23$$
$$6x = 23 - 2 = 21$$
$$x = 3.5$$

Hence Weight of cement bag = 3.5 kg
Weight of sand bag = 1 kg

-:8.10:-

Jack and Jill's ages add up to 30 years. In 12 years time Jack will be twice as old as Jill is now. How old are Jack and Jill now?

SOLUTION

Let x = The age of Jack and
y = The age of Jill.

Thus

$$x + y = 30$$

After 12 years, the Jack will be 2x, if Jill's age will be y. The sum of ages of Jack and Jill after 12 years will be 54. Thus

$$2x + y = 54$$

Again we write equations as:

$$x + y = 30 \text{ .... } (1)$$
$$2x + y = 54 \text{ .... } (2)$$

To eliminate y, subtracting eq (2) from eq (1), we get

$$-x = -24$$
$$x = 24$$

For y, putting x = 24 in eq (1), we get

$$24 + y = 30$$
$$y = 30 - 24$$
$$y = 6$$

Hence Now Jack's age = 24 years and
Jill's age = 6 years

-:8.11:-

The age of a man 10 years hence was 4 times the age of his son. Father's age after 10 years will be twice the age of the son. Find their present ages.

SOLUTION

Let x = The present age of Father and
y = The present age of Son

Thus 10 years before the age of Father = x − 10
10 years before the age of Son = y − 10
After 10 years the age of Father = x + 10
After 10 years the age of Son = y + 10

Now According to given conditions

$$x - 10 = 4(y - 10) \quad \text{(Before 10 years)}$$
$$x - 10 = 4y - 40$$
$$x - 4y = -40 + 10$$
$$x - 4y = -30$$

and

$$(x + 10) = 2(y + 10) \quad \text{(After 10 years)}$$
$$x + 10 = 20 - 10$$
$$x - 2y = 10$$

Thus

$$x - 4y = -30 \text{ .... } (1)$$
$$x - 2y = 10 \text{ .... } (2)$$

For y. subtracting equation (2) from equation (1), we get

$$x-4y=-30$$
$$\underline{-x+2y=-10}$$
$$-2y=-40$$
$$y=20$$

For x. putting y = 20 in eq (2), we get

$$x-2(20)=10$$
$$x-40=10$$
$$x=10+40=50$$

Hence The present age of Father = 50 years and
The present age of Son = 20 years

-:8.12:-

The age of a man 5 years ago was seven times the age of his son. The ratio of their ages after 5 years will be 3:1. Find their present ages.

SOLUTION

Let x = The present age of Father and
y = The present age of Son
5 years ago
Father's age = x − 5 and Son's age = y − 5
5 years after
Father age = x + 5 and Son's age = y + 5

According to given conditions

$$(x-5)=7(y-5) \quad \text{(5 years ago)}$$
$$x-5=7y-35$$
$$x-7y=-35+5$$
$$x-7y=-30$$

and

$$(x+5)=3(y+5) \quad \text{(After 5 years)}$$
$$x+5=3y+15$$
$$x-3y=15-5$$
$$x-3y=10$$

Thus

$$x-5y=-30 \quad \ldots\ldots\ldots\ldots(1)$$
$$x-3y=10 \quad \ldots\ldots\ldots\ldots(2)$$

To eliminate x, subtracting equation (2) from equation (1), we get

$$\begin{array}{r} x-7y=-30 \\ x-3y=10 \\ \hline -4y=-40 \\ y=10 \end{array}$$

For x, putting y = 10 in eq (2), we get

$$x-3(10)=10$$
$$x-30=10$$
$$x=10+30=40$$

Hence The present age of Father = x = 40 years and
The present age of Son = y = 10 years

-:8.13:-

A library is buying a total of 100 books for Rs. 8460. Some of the books cost Rs. 65 each, and the remainder cost Rs. 100 per book. How may books of each price are they buying?

SOLUTION

Let x = No of Books of Rs. 65 Cost
y = No of Books of Rs. 100 Cost

$$x+y=100$$

Total Cost of x Books @ Rs. 65 = 65x and
Total Cost of y Books @ Rs. 100 = 100y
Total Cost of x Books + Total Cost of y Book = Total Cost of Books

$$65x+100y=\text{Rs. }8460$$

Thus we have,

$$x+5y=100 \quad \ldots\ldots\ldots\ldots\ldots(1)$$
$$65x+100y=8460 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

For y, multiplying equation (1) by 65 and subtracting equation (2) from it.

$$\begin{array}{r} 65x+65y=6500 \\ 65x+100y=8460 \\ \hline -35y=-1960 \\ y=56 \end{array}$$

For x, putting y = 65 in eq (1), we get

$$x + 56 = 100$$

$$x = 100 - 56$$

$$x = 44$$

Hence No of Books @ Rs. 65 = 44 Books and
No of Books @ Rs. 100 = 56 Books

-:8.14:-

If a bird flies 40 km/hr with the wind, but only 10 km/hr against the wind, what is the rate of bird in still air? What is the rate of wind?

SOLUTION

Let x = Rate of Bird and
y = Rate of Wind
Rate of Bird + Rate of Wind = Rate with the Wind
Rate of Bird – Rate of Wind = Rate against the Wind

$$x + y = 40 \quad \text{...............(1)}$$

$$x - y = 10 \quad \text{...............(2)}$$

Adding equation (1) and (2), we get

$$x + y = 40$$

$$\underline{x - y = 10}$$

$$2x = 50$$

$$x = 25$$

For y, putting x = 25 in eq (1), we get

$$25 + y = 40$$

$$y = 40 - 25$$

$$y = 15$$

Hence Rate of Bird = x = 25 km/h and
Rate of Wind = y = 15 km/h

-:8.15:-

If an airplane can travel 370 km/hr with the wind, but only 320 km/hr against the wind, what is the plane's speed is still air?

SOLUTION

Let x = Rate of Airplane
y = Rate of Wind

Rate of Airplane + Rate of Wind = Rate with the Wind

Rate of Airplane – Rate of Wind = Rate against the Wind

$$x + y = 370 \quad \text{..............(1)}$$

$$x - y = 320 \quad \text{..............(2)}$$

Adding equation (1) and equation (2), we get

$$x + y = 370$$
$$x - y = 320$$
$$\overline{\quad 2x = 690 \quad}$$
$$x = 345 \text{ mph}$$

Hence The Airplane's speed in still air = x = 345 mph

-:8.16:-

The price of daily newspaper "JANG" on Sunday difference from its price on the other week days. A family paid Rs. 248 for 30 days newspaper including four Sundays. Next month, 30 days newspaper, including five Sundays cost Rs. 250. Find the price of the newspaper on Sundays and its prices on other week days?

SOLUTION

Let x = Price of Newspaper on Sunday

y = Price of Newspaper on the other day

Thus, According to given condition

$$4x + 26y = 248 \quad \text{...............(1)}$$

$$5x + 25y = 250 \quad \text{..............(2)}$$

To eliminate x, multiplying equation (1) by 5 and equation (2) by 4, subtracting

$$20x + 130y = 1240$$
$$\underset{-}{}20x \underset{-}{+} 100y \underset{-}{=} 1000$$
$$\overline{\quad 30y = 240 \quad}$$
$$y = 8$$

For x, putting y = 8 in eq (1), we get

$$4x + 26(8) = 248$$
$$4x + 208 = 248$$
$$4x = 248 - 208$$
$$x = 10$$

Hence Price of Sunday Newspaper = x = Rs. 10 and

Price of other days Newspaper = y = Rs. 8

-:8.17:-

A bus hire firm uses large coaches which take 40 people and small coaches which take 25 people each. If 31 coaches are to be used, how many large coaches and how many small coaches will be needed to carry 1000 people.

SOLUTION

Let x = The number of large coaches

Y = The number of small coaches

Thus

$$x + y = 31 \text{ .... } (1)$$

$$40x + 25y = 1000 \text{ .... } (2)$$

To eliminate y, multiplying eq (1) by 25 and subtracting eq (2) from it. We get

$$\begin{array}{r} 25x + 25y = 775 \\ -\ \underline{40x \pm 25y = -1000} \\ -15y = -225 \\ x = 15 \end{array}$$

For y, putting $x = 15$ in eq (1), we get

$$15 + y = 31$$

$$y = 31 - 15 = 16$$

$$y = 16$$

Hence No. of large coaches = $x = 15$

No. of small coaches = $y = 16$

-:8.18:-

Hassan invested a total of Rs. 2000, part at 10% and the remainder at 11%. His income from the two investments is Rs. 212. How much did Hassan invest at each rate?

SOLUTION

Let x = Represents the amount invested at 10% and

y = Represents the amount invested at 11%, then

$$x + y = 2000 \text{ .... } (1)$$

$$0.10x + 0.11y = 212 \text{ .... } (2)$$

Multiplying equation (1) by $-11$and equation (2) by 100, and adding them, we get

$$-11x - 11yy = 22000$$
$$10x + 11y = 21200$$
$$\overline{\quad -x = -800 \quad}$$
$$x = 800$$

For y, putting x = 800 in eq (1), we get

$$800 + y = 2000$$
$$y = 2000 - 800$$
$$y = 1200$$

Hence Investment at 10% = x = Rs. 800 and
Investment at 11% = y = Rs. 1200

-:8.19:-

A man drives x hours at 50 km per hour, and y hours at 60 km per hour. He drives for a total 6 hours.. For how long does he drive at each speed if he drives 330 kms?

SOLUTION

Here Total Time = 6 hours

Total Distance Travelled = 330 kms

Distance covered by 50 km/h = Time x Rate = 50x

Distance covered by 60 km/h = Time x Rate = 60y

According to given conditions, we have

$$x + y = 6 \quad \text{(Total Time)} \quad \ldots\ldots\ldots\ldots\ldots(1)$$
$$50x + 60y = 330 \quad \text{(Total Distance} \ldots\ldots\ldots\ldots\ldots(2)$$

Multiplying equation (1) by –50 and adding equation (2) to it, we get

$$-50x - 50y = -300$$
$$50x + 60y = 330$$
$$\overline{\quad 10y = 30 \quad}$$
$$y = 3$$

For x, putting y = 3 in eq (1), we get

$$x + 3 = 6$$
$$x = 6 - 3$$
$$x = 3$$

Hence Time for 50 km/h speed = x = 3 hrs and

Time for 60 km/h speed = y = 3 hrs

-:8.20:-

A hotel rents double rooms at Rs. 150 per day and single rooms at Rs. 100 per day. If a total of 50 rooms were rented one days for Rs. 6750, how many rooms of each kind were rented?

SOLUTION

Let x = Number of Rooms Double rented

Y = Number of Rooms Single rented

Thus

$$x + y = 50 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$150x + 100y = 6750 \ldots\ldots\ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −100 and adding equation (2) to it, we get

$$\begin{array}{r} -100x - 100y = -5000 \\ 150x + 100y = 6750 \\ \hline 50x = 1750 \\ x = 35 \end{array}$$

For y, putting x = 35 in eq (1), we get

$$35 + y = 50$$

$$y = 50 - 30$$

$$y = 15$$

Hence No of Rooms Double rented = x = 35 and

No of Rooms Single rented = y = 15

-:8.21:-

A 10% salt solution is to be mixed with a 20% salt solution to produce 20 gallons of 17.5% salt solution. How many gallons of the 10% solution and how many gallons of the 20% solution will be needed?

SOLUTION

Let x = The number of Gallons of the 10% salt solution, and

y = The number of Gallon's of the 20% salt solution

The total amount is

$$x + y = 20 \text{ Gallon} \ldots\ldots\ldots (1)$$

and

$$10\%x + 20\%y = 17.5\% (20)$$

$$0.10x + 0.20y = 0.175 (20)$$

$$0.10x + 0.20y = 3.5 \ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −20 and equation (2) by 100 and adding to it, we get

$$-20x - 20y = -400$$

$$10x + 20y = 350$$

$$-10x = -50$$

$$x = 5$$

For y, putting x = 5 in eq (1), we get

$$5 + y = 20$$

$$y = 20 - 5$$

$$y = 15$$

Hence The number of Gallons of 10% salt = x = 5 gallons and

The number of Gallons of 20% salt = y = 15

-:8.22:-

One solution contains 50% alcohol and another solution contains 80% alcohol. how many liters of each solution should be mixed to make 21 liters of 70% solution.

SOLUTION

Let x = The number of Liters of 50% Alcohol, and

y = The number of Liters of 80% Alcohol

Then

$$x + y = 21 \text{ Liters} \ldots\ldots\ldots (1)$$

According to given conditions, we have

$$0.50x + 0.80y = 0.70\ (21)$$

$$0.50x + 0.80y = 14.7 \ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −80 and equation (2) by 100 and adding to it, we get

$$-80x - 80y = -1680$$
$$\underline{50x + 80y = 1470}$$
$$-30x = -210$$
$$x = 7 \text{ liters}$$

For y, putting x = 7 in eq (1), we get

$$7 + y = 21$$
$$y = 21 - 7$$
$$y = 14 \text{ liters}$$

Hence The No of Liters of 50% Alcohol = x = 7 Liters and
The no of Liters of 80% Alcohol = y = 14 Liters

EXERCISE NO. 9

-:9.1:-

Convert the following number from base ten to base two. 64

SOLUTION

2	64	Reminder
2	28	 0
2	16	 0
2	8	 0
2	4	 0
2	2	 0
	1	 0

$(1000000)_2$

-:9.2:-

Convert the following number from base ten to base two. 125

SOLUTION

2	125	Reminder
2	62	 1
2	31	 0
2	15	 1
2	7	 1
2	3	 1
	1	 1

$(1111101)_2$

-:9.3:-

Convert the following number from base ten to base two. 586

SOLUTION

2	586	Reminder
2	293	 0
2	146	 1
2	73	 0
2	36	 1
2	18	 0
2	9	 0
2	4	 1
2	2	 0
	1	 0

$(1001001010)_2$

-:9.4:-

Convert the following number from base ten to base two.

445

SOLUTION

2	445	Reminder
2	222	 1
2	111	 0
2	55	 1
2	27	 1
2	13	 1
2	6	 1
2	3	 0
	1	 1

$(110111101)_2$

-:9.6:-

Convert the following number from base ten to base two.

1551

SOLUTION

2	1551	Reminder
2	775	 1
2	387	 1
2	193	 1
2	96	 1
2	48	 0
2	24	 0
2	12	 0
2	6	 0
2	3	 0
	1	 1

$(11000001111)_2$

-:9.7:-

Convert the following number from base ten to base two.

808

SOLUTION

2	808	Reminder
2	404	 0
2	202	 0
2	101	 0
2	50	 1
2	25	 0
2	12	 1
2	6	 0
2	3	 0
	1	 1

$(1100101000)_2$

-:9.8:-

Convert the following number from base ten to base two.

2525

SOLUTION

2	2525	Reminder
2	1262	 1
2	631	 0
2	315	 1
2	157	 1
2	78	 1
2	39	 0
2	19	 1
2	9	 1
2	4	 1
2	2	 0
	1	 0

$(100111011101)_2$

-:9.9:-

Convert the following number from base two to base ten.

10101

SOLUTION

$(10101)_2$

2^4	2^3	2^2	2^1	2^0
1	0	1	0	1

OR

16	8	4	2	1
1	0	1	0	1

$(10101)_2 = (1\times16) + (0\times8) + (1\times4) + (0\times2) + (1\times1)$

$= 16 + 0 + 4 + 0 + 1 = 16 + 4 + 1 = 21$

-:9.10:-

Convert the following number from base two to base ten.

$(11111)_2$

SOLUTION

$(11111)_2$

2	2^3	2^2	2^1	2^0
1	1	1	1	1

OR

16	8	4	2	1
1	1	1	1	1

$(11111)_2 = (1\times16) + (1\times8) + (1\times4) + (1\times2) + (1\times1)$

$= 16 + 8 + 4 + 2 + 1 = 31$

-:9.11:-

Convert the following number from base two to base ten.

$(100000)_2$

SOLUTION

$(100000)_2$

2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	0	0	0

OR

32	16	8	4	2	1
1	0	0	0	0	0

$(100000)_2 = (1\times32) + (0\times16) + (0\times8) + (0\times4) + (0\times2) + (0\times1)$

$= 32 + 0 + 0 + 0 + 0 + 0 = 32$

-:9.12:-

Convert the following number from base two to base ten.

$(101001)_2$

SOLUTION

$(1011001)_2$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	0	1

OR

64	32	16	8	4	2	1
1	0	1	1	0	0	1

$(1011001)_2 = (1\text{x}64)+(0\text{x}32)+(1\text{x}16)+(1\text{x}8)+(0\text{x}4)+(0\text{x}2)+(1\text{x}1)$

$= 64 + 0 + 16 + 8 + 0 + 0 + 1$

$= 64 + 16 + 8 + 1 = 89$

-:9.13:-

Convert the following number from base two to base ten.

$(11100111)_2$

SOLUTION

$(11100111)_2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	0	0	1	1	1

OR

128	64	32	16	8	4	2	1
1	1	1	0	0	1	1	1

$(11100111)_2 = (1\text{x}128) + (1\text{x}64) + (1\text{x}32) + (0\text{x}16) + (0\text{x}8) + (1\text{x}4)$
$+ (1\text{x}2) + (1\text{x}1)$

$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 1$

$= 128 + 64 + 32 + 4 + 2 + 1 = 231$

-:9.14:-

Convert the following number from base two to base ten.

$(1001001001)_2$

SOLUTION

$(1001001001)_2$

2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	0	1	0	0	1

OR

512	256	128	64	32	16	8	4	2	1
1	0	0	1	0	0	1	0	0	1

$(1001001001)_2 = (1\times512)+(0\times256)+(0\times128)+(1\times64)+(0\times32)+(0\times16)$
$+(1\times8)+(0\times4)+(0\times2)+(1\times1)$
$= 512+0+0+64+0+0+8+0+0+1$
$= 512+64+8+1 = 585$

-:9.15:-

Convert the following number from base two to base ten.

$(111111000)_2$

SOLUTION

$(111111000)_2$

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	1	1	1	0	0	0

OR

256	128	64	32	16	8	4	2	1
1	1	1	1	1	1	0	0	0

$(111111000)_2 = 1\times256)+(1\times128)+(1\times64)+(1\times32)+(1\times16)+(1\times8)+(0\times4)$
$+(0\times2)+(0\times1)$
$= 256+128+64+32+16+8+0+0+0$
$=256+128+64+32+16+8 = 504$

Adding in base two check by converting to base ten.

-:9.16:-

```
  1011
  1011
------
110000
```

-:9.17:-

```
 1101
 1101
-----
11000
```

-:9.18:-

```
 11010
 10111
------
110001
```

-:9.19:-

```
 101111
 111011
 101101
-------
10010111
```

-:9.20:-

```
   1111
   1111
   1111
 ------
 101101
```

-:9.21:-

```
   11011
   11100
   10111
 -------
 1001110
```

-:9.22:-

```
   1111111
   1110101
   1101011
   1110100
 ---------
 111010011
```

-:9.23:-

```
   1110001
   1011101
   1001101
   1101010
 ---------
 110000101
```

-:9.24:-

```
   11011101
   10111011
   11100111
   10111011
 ----------
 1100111010
```

-:9.25:-

```
   1011010
 -  100101
 ---------
    110101
```

-:9.26:-

```
   1101011
 -  111101
 ---------
    101110
```

-:9.27:-

```
   110000
 -  11011
 --------
    10101
```

-:9.28:-

```
   11111111
 -  1111111
 ----------
   10000000
```

-:9.29:-

```
   11011011
 -   101011
 ----------
   10110000
```

-:9.30:-

```
    111011011
 -  101010000
 ------------
     10001011
```

-:9.31:-

```
      101
      111
 --------
      101
     101x
    101xx
 --------
   100011
```

-:9.32:-

```
     1011
      111
  -------
     1011
    1011x
   1011xx
  -------
  1001101
  -------
```

-:9.33:-

```
     11111
      1100
 ---------
     00000
    00000x
   11111xx
  11111xxx
 ---------
 101110100
 ---------
```

-:9.34:-

```
      110111
       10101
 -----------
      110111
     000000x
    110111xx
   000000xxx
  110111xxxx
 -----------
 10010000011
 -----------
```

-:9.35:-

```
     11111
      1001
 ---------
     11111
    00000x
   00000xx
  11111xxx
 ---------
 100010111
 ---------
```

-:9.36:-

```
        1111011
         111011
 --------------
        1111011
       1111011x
      0000000xx
     1111011xxx
    1111011xxxx
   1111011xxxxx
 --------------
 11100001011001
 --------------
```

-:9.37:-

```
      111111
       11111
 -----------
      111111
     111111x
    111111xx
   111111xxx
  111111xxxx
 -----------
 11110100001
 -----------
```

-:9.38:-

$(11001)_2 \div (101)_2$

$$\begin{array}{r} 101 \\ 101\overline{)11001} \\ \underline{101} \\ 101 \\ \underline{101} \\ x \end{array}$$

$(101)_2$

-:9.39:-

$(110110)_2 \div (1001)_2$

$$\begin{array}{r} 110 \\ 1001\overline{)110110} \\ \underline{1001} \\ 1001 \\ \underline{1001} \\ x \end{array}$$

$(110)_2$

-:9.40:-

$(1110111)_2 \div (11010)_2$

$$\begin{array}{r} 10 \\ 11010\overline{)1110111} \\ \underline{11010} \\ 111 \end{array}$$

$(10)_2$ with remainder $(111)_2$

-:9.41:-

$(1011010110)_2 \div (10110)_2$

$$\begin{array}{r} 100001 \\ 10110\overline{)1011010110} \\ \underline{10110} \\ 10110 \\ \underline{10110} \\ x \end{array}$$

$(100001)_2$

-:9.42:

$\frac{1}{2}$

$1 = 1_2$

$2 = 10_2$

$\frac{1}{2} = \frac{1_2}{10_2}$

$$\begin{array}{r} 0.1 \\ 10\overline{)10} \\ \underline{10} \\ x \end{array}$$

$(0.1)_2$

Alternative method

$\frac{1}{2} = 0.5$

Multiplying by two

$$\begin{array}{lr} & 0.5 \\ & 2 \\ \hline 1 & \ldots\ldots 1.0 \end{array}$$

$(0.1)_2$

-:9.43:-

$\frac{1}{4}$

$1 = 1_2$

$4 = 100_2$

$$\begin{array}{r|l} & 0.01 \\ 100 & 100 \\ & 100 \\ \hline & x \end{array}$$

Hence $\frac{1}{4} = (0.01)_2$

Alternative method

$\frac{1}{4} = 0.25$

Multiplying by two

$$\begin{array}{lr} & 0.25 \\ & 2 \\ \hline 0 & ....0.50 \\ & 2 \\ \hline 1 & ......1.00 \end{array}$$

-:9.44:-

$\frac{3}{4}$

$3 = 11_2$ and

$4 = 100_2$

$$\begin{array}{r|l} & 0.11 \\ 100 & 110 \\ & 100 \\ \hline & 100 \\ & 100 \\ \hline & x \end{array}$$

Hence $\frac{3}{4} = (0.11)_2$

Alternative method

$\frac{3}{4} = (0.75)_2$

$$\begin{array}{lr} & 0.75 \\ & 2 \\ \hline 1 & ....0.50 \\ & 2 \\ \hline 1 & ......1.00 \end{array}$$

-:9.45:-

$\frac{24}{60} = \frac{2}{5}$

$\frac{2}{5} = \frac{(100)_2}{(101)_2}$

$$\begin{array}{r|l} 101 & 1000\ (.110011001100 11....... \\ & 101 \\ \hline & 1000 \\ & 101 \\ \hline & 110 \\ & 101 \\ \hline & 1 \end{array}$$

$\frac{24}{60} = (0.110011001100 11.....)_2$

-:9.46:-

$$\frac{72}{120} = \frac{3}{5}$$

$$\frac{3}{5} = \frac{(11)_2}{(101)_2}$$

$$\begin{array}{r|l} 101 & 110 \quad (.100110011\ldots\ldots \\ & \underline{101} \\ & 1000 \\ & \underline{101} \\ & 1000 \\ & \underline{101} \\ & 110 \\ & \underline{101} \\ & 1 \end{array}$$

$$\frac{72}{120} = (0.1001100110011\ldots\ldots)_2$$

-:9.47:-

Convert each of the following binary numbers in decimal.

$(111.11)_2$

SOLUTION

2^2	2^1	2^0	2^{-1}	2^{-2}
1	1	1	1	1

OR

4	2	1	½	¼
1	1	1	1	1

$(111.11)_2 = (1\text{x}4) + (1\text{x}2) + (1\text{x}1) + (1\text{x}½) + (1\text{x}¼)$

$= 4 + 2 + 1 + ½ + ¼ = 4 + 2 + 1 + 0.5 + 0.25 = 7.75$

-:9.48:-

Convert each of the following binary numbers in decimal.

$(1110.111)_2$

SOLUTION

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	1	0	1	1	1

OR

4	2	1	½	¼	$^1/_8$
1	1	0	1	1	1

$(110.111)_2 = (1x4)+(1x2)+(0x1)+(1x½)+(1x¼)+(1x^1/_8)$

$= 4 + 2 + 0 + ½ + ¼ + ^1/_8$

$= 4 + 2 + 0.5 + 0.25 + 0.125 = 6.875$

-:9.49:-

Convert each of the following binary numbers in decimal.

$(1111.111)_2$

SOLUTION

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	1	1	1	1	1	1

OR

8	4	2	1	½	$^1/_4$	$^1/_8$
1	1	1	1	1	1	1

$(1111.111)_2 = (1x8)+(1x4)+(1x2)+(1x1)+(1x½)+(1x¼)+(1x^1/_8)$

$= 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125 = 15.875$

-:9.50:-

Convert each of the following binary numbers in decimal.

$(101010.001)_2$

SOLUTION

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	0	1	0	1	0	0	0	1

OR

32	16	8	4	2	1	½	$^1/_4$	$^1/_8$
1	0	1	0	1	0	0	0	1

$(101010.001)_2 = (1x32)+(0x16)+(1x8)+(0x4)+(1x2)+(0x1)+(0x½)$

$+(0x¼)+(1x^1/_8)$

$= 32 + 0 + 8 + 0 + 2 + 0 + 0 + 0 + ^1/_8$

$= 32 + 8 + 2 + 0.125 = 42.125$

EXERCISE NO. 10

-:10.1:-

State the order of each of the following matrices:

i) $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 2 \end{bmatrix}$

ii) $B = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 4 \\ 3 & 6 & 3 \end{bmatrix}$

iii) $C = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 6 & 7 & 8 \\ 3 & 5 & 2 & 1 \end{bmatrix}$

iv) $D = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 3 & 3 \\ 6 & 5 & 4 \\ 8 & 7 & 5 \end{bmatrix}$

SOLUTION

i) The order of the matrix is 2×3
ii) The order of the matrix is 3×3
iii) The order of the matrix is 3×4
iv) The order of the matrix is 4×3

-:10.2:-

Write the general 2x2 matrix, using double subscript notation.

SOLUTION

The general 2×2 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

-:10.3:-

Write the general 4x4 matrix using double subscript notation.

SOLUTION

The general 4×4 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times 4}$$

-:10.4:-

The dimension of A is 3 x 4, the dimension of B is 4 x 1., the dimension of C is 1 x 3, the dimension of D is 4 x 3 and the dimension of E is 3 x 3. Determine the dimension of

(a)	AB	(b)	BA	(c)	CA	(d)	AD
(e)	DA	(f)	BCE	(g)	CEA	(h)	ABCD
(i)	DABCE						

SOLUTION

a) AB = (3 x 4) (4 x 1) = 3 x 1

b) BA = (4 x 1) (3 x 4) = Does not exist

c) CA = (1 x 3) (3 x 4) = 1 x 4

d) AD = (3 x 4) (4 x 3) = 3 x 3

e) DA = (4 x 3) (3 x 4) = 4 x 4

f) BCE = (4 x 1) (1 x 3) (3 x 3) = 4 x 3

g) CEA = (4 x 1) (1 x 3) (3 x 3) = 4 x 3

h) ABCD = (3 x 4) (4 x 1) (1 x 3) (4 x 3) = Does not exist

i) DABCE = (4 x 3) (3 x 4) (4 x 1) (1 x 3) (3 x 3) = 4 x 3

-:10.5:-

If

$$A=\begin{bmatrix}1 & 4\\ 3 & 2\\ 2 & 5\end{bmatrix} \quad B=\begin{bmatrix}6 & 5\\ 2 & 6\\ 1 & 1\end{bmatrix}$$

Find A+B, B+A, A-B and B-A

SOLUTION

$$A=\begin{bmatrix}1 & 4\\ 3 & 2\\ 2 & 5\end{bmatrix} \quad B=\begin{bmatrix}6 & 5\\ 2 & 6\\ 1 & 1\end{bmatrix}$$

$$A+B=\begin{bmatrix}1 & 4\\ 3 & 2\\ 2 & 5\end{bmatrix}+\begin{bmatrix}6 & 5\\ 2 & 6\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1+6 & 4+5\\ 3+2 & 2+6\\ 2+1 & 5+1\end{bmatrix}=\begin{bmatrix}7 & 9\\ 5 & 8\\ 3 & 6\end{bmatrix}$$

$$B+A=\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}+\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}=\begin{bmatrix}6+1&5+4\\2+3&6+2\\1+2&1+5\end{bmatrix}=\begin{bmatrix}7&9\\5&8\\3&6\end{bmatrix}$$

$$A-B=\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}+\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}=\begin{bmatrix}1-6&4-5\\3-2&2-6\\2-1&5-1\end{bmatrix}=\begin{bmatrix}-5&-1\\1&-4\\1&4\end{bmatrix}$$

$$B-A=\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}+\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}=\begin{bmatrix}6-1&5-4\\2-3&6-2\\1-2&1-5\end{bmatrix}=\begin{bmatrix}-5&-1\\1&-4\\-1&-4\end{bmatrix}$$

-:10.6:-

If $A=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}, B=\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$

Then find A + B and A – B

SOLUTION

$$A+B=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}+\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$$

$$=\begin{bmatrix}x-2y+x+2y&y+x-y&x+z+x-z\\x-x+3y&x-y+x+y&4x+y+3x-y\\4x+3y-4x&-x+y+x-y&3y+2x-3y\end{bmatrix}$$

$$=\begin{bmatrix}2x&x&2x\\3y&2x&7x\\3y&0&2x\end{bmatrix}$$

Now

$$A-B=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}-\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$$

$$A - B = \begin{bmatrix} x - 2y - x - 2y & y - x + y & x + z - x + z \\ x + x - 3y & x - y - x - y & 4x + y - 3x + y \\ 4x + 3y + 4x & x + y - x + y & 3y - 2x + 3y \end{bmatrix}$$

$$= \begin{bmatrix} -4x & -x + 2y & 2z \\ 2x - 3y & -2y & x + 2y \\ 8x + 3y & -2x + 2y & -2x + 6y \end{bmatrix}$$

-:10.7:-

If $A = \begin{bmatrix} a & a + b \\ b - c & -c \\ a + c & b + c \end{bmatrix}, B = \begin{bmatrix} -a + b & -b \\ c & b + c \\ -a & -c \end{bmatrix}$

Then find A + B and A – B

SOLUTION

$$A + B = \begin{bmatrix} a & a + b \\ b - c & -c \\ a + c & b + c \end{bmatrix} + \begin{bmatrix} -a + b & -b \\ c & b + c \\ -a & -c \end{bmatrix}$$

$$= \begin{bmatrix} a - a + b & a + b - b \\ b - c + c & -c + b + c \\ a + c - a & b + c - c \end{bmatrix} = \begin{bmatrix} b & a \\ b & b \\ c & b \end{bmatrix}$$

Now

$$A - B = \begin{bmatrix} a & a + b \\ b - c & -c \\ a + c & b + c \end{bmatrix} - \begin{bmatrix} -a + b & -b \\ c & b + c \\ -a & -c \end{bmatrix}$$

$$= \begin{bmatrix} a + a - b & a + b + b \\ b - c - c & -c - b - c \\ a + c + a & b + c + c \end{bmatrix} = \begin{bmatrix} 2a - b & a + 2b \\ b - 2c & -b - 2c \\ 2a + c & b + 2c \end{bmatrix}$$

-:10.8:-

Find A if 2A + 3B = C where

$$\mathbf{B}=\begin{bmatrix}1 & 2\\ 0 & 4\\ 4 & 0\end{bmatrix} \text{ and } \mathbf{C}=\begin{bmatrix}3 & -1\\ 2 & 0\\ -1 & -1\end{bmatrix}$$

Hint: Since 2A + 3B = C, it follows that 2A = C – 2B and

$$\mathbf{A}=\frac{1}{2}(\mathbf{C}-3\mathbf{B})$$

SOLUTION

We know that 2A + 3B = C

2A = C – 3B and A = ½ (C – 3B)

$$3B=3\begin{bmatrix}1 & 2\\ 0 & 4\\ 4 & 0\end{bmatrix}=\begin{bmatrix}3 & 6\\ 0 & 12\\ 12 & 0\end{bmatrix}$$

$$(C-3B)=\begin{bmatrix}3 & -1\\ 2 & 0\\ -1 & -1\end{bmatrix}-\begin{bmatrix}3 & 6\\ 0 & 12\\ 12 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3-3 & -1-6\\ 2-0 & 0-12\\ -1-12 & -1-0\end{bmatrix}=\begin{bmatrix}0 & -7\\ 2 & -12\\ -13 & -1\end{bmatrix}$$

So

$$A=\frac{1}{2}(C-3B)=\frac{1}{2}\begin{bmatrix}0 & -7\\ 2 & -12\\ -13 & -1\end{bmatrix}=\begin{bmatrix}0 & -\frac{7}{2}\\ 1 & -6\\ -\frac{13}{2} & -\frac{1}{2}\end{bmatrix}=\begin{bmatrix}0 & -3.5\\ 1 & -6\\ -6.5 & -0.5\end{bmatrix}$$

-:10.9:-

Find B if –A + 2B = 6C, where

$$\mathbf{A}=\begin{bmatrix}1 & 2 & -1\\ 3 & 0 & 1\\ 1 & 1 & 1\end{bmatrix} \text{ and } \mathbf{C}=\begin{bmatrix}2 & -1 & 7\\ 3 & 0 & 0\\ 4 & 1 & -5\end{bmatrix}$$

Hint: Since –A + 2B = 6C, it follows that 2B = A + 6C and

B = ⅓ (A + 3C)

SOLUTION

Since –A + 2B = 6C, it follows 2B = A + 6C and B = ½ (A + 6C)

$$6C = 6\begin{bmatrix}2 & -1 & 7\\ 3 & 0 & 0\\ 4 & 1 & -5\end{bmatrix} = \begin{bmatrix}12 & -6 & 42\\ 18 & 0 & 0\\ 24 & 6 & -30\end{bmatrix}$$

$$A + 6C = \begin{bmatrix}1 & 2 & -1\\ 3 & 0 & 1\\ 1 & 1 & 1\end{bmatrix} + \begin{bmatrix}12 & -6 & 42\\ 18 & 0 & 0\\ 24 & 6 & -30\end{bmatrix} = \begin{bmatrix}13 & -4 & 41\\ 21 & 0 & 1\\ 25 & 7 & -29\end{bmatrix}$$

$$B = \frac{1}{2}(A + 6C) = \frac{1}{2} = \begin{bmatrix}13 & -4 & 41\\ 21 & 0 & 1\\ 25 & 7 & -29\end{bmatrix} = \begin{bmatrix}\frac{13}{2} & -2 & \frac{41}{2}\\ \frac{21}{2} & 0 & \frac{1}{2}\\ \frac{25}{2} & \frac{7}{2} & -\frac{29}{2}\end{bmatrix}$$

$$= \begin{bmatrix}7.5 & -1 & 20.5\\ 10.5 & 0 & 0.5\\ 12.5 & 3.5 & -14.5\end{bmatrix}$$

-:10.10:-

Write the transpose of the following matrices.

(i) $A = \begin{bmatrix}3 & 2 & 6\\ 1 & 4 & 2\\ 6 & 0 & 7\end{bmatrix}$ **(ii)** $A = \begin{bmatrix}3 & 2\\ 4 & 6\\ 7 & 2\end{bmatrix}$

SOLUTION

(i) $A = \begin{bmatrix}3 & 2 & 6\\ 1 & 4 & 2\\ 6 & 0 & 7\end{bmatrix}$

Interchange the rows and columns

$$A^t = \begin{bmatrix}3 & 1 & 6\\ 2 & 4 & 0\\ 6 & 2 & 7\end{bmatrix}$$

(ii) $$A = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 7 & 2 \end{bmatrix}$$

Interchange the rows and columns

$$A^t = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 6 & 2 \end{bmatrix}$$

-:10.11:-

If A is matrix, what is $[A^t]^t$

SOLUTION

If A is the matrix then its transpose is A^t. The transpose of A^t is again the original matrix i.e. $[A^t]^t = A$

-:10.12:-

Find x, y, z and w from the following matrices.

a) $$\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$ **b)** $$\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$$

SOLUTION

(a)

$$\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

The two matrices are equal. In the equality of matrices the corresponding elements are same. Hence from the above matrices we find x=3, y=3, z=3 and w=4

(b)

$$\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$$

Here x = 4, y = 1, z = 1 and w = 2

-:10.13:-

Solve for x, y and z if

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

SOLUTION

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

We first add the two matrices of left hand side

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 2+2x & y-y \\ y+3y & z-4z \end{bmatrix} = \begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix}$$

Hence

$$\begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

From the two equal matrices, we have

$$3x = 6 \qquad 4y = 8 \qquad -3z = 9$$
$$x = 2 \qquad y = 2 \qquad z = -3$$

-:10.14 :-

If

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find the 3A+4B = 2C

SOLUTION

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

$$3A + 4B - 2C$$

$$3A = 3\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 6 & -15 & 3 \\ 9 & 0 & -12 \end{bmatrix}$$

$$4B = 4\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -8 & -12 \\ 0 & -4 & 20 \end{bmatrix}$$

$$2C = 2\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -4 \\ 2 & -2 & -2 \end{bmatrix}$$

$$3A + 4B = \begin{bmatrix} 6 & -15 & 3 \\ 9 & 0 & -12 \end{bmatrix} + \begin{bmatrix} 4 & -8 & -12 \\ 0 & -4 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 6+4 & -15-8 & 3-12 \\ 9+0 & 0-4 & -12+20 \end{bmatrix} = \begin{bmatrix} 10 & -23 & -9 \\ 9 & -4 & 8 \end{bmatrix}$$

$$3A + 4B - 2C = \begin{bmatrix} 10 & -23 & -9 \\ 9 & -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 2 & -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 10+0 & -23-2 & -9-4 \\ 9-2 & -4+2 & 8+2 \end{bmatrix} = \begin{bmatrix} 10 & -25 & -13 \\ 7 & -2 & 10 \end{bmatrix}$$

-:10.15:-

Find XZ, ZX and XY, if possible for

$$X = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, Y = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, Z = \begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$$

SOLUTION

$$XZ = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2(2) & 2(1) & 2(4) \\ 1(2) & 1(1) & 1(4) \\ 3(2) & 3(1) & 3(4) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 8 \\ 2 & 1 & 4 \\ 6 & 3 & 12 \end{bmatrix}$$

$$ZX = \begin{bmatrix} 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2(2) + 1(1) + 4(3) \end{bmatrix} = \begin{bmatrix} 4+1+12 \end{bmatrix} = 17$$

$$XY = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \text{Does not exist}$$

-:10.16:-

Verify that AB = AC, but that B ≠ C, for

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 5 & 6 \\ -1 & -3 \end{bmatrix}$$

SOLUTION

$$AB=\begin{bmatrix}2 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}2(1)+4(1) & 2(2)+4(-1)\\1(1)+2(1) & 1(2)+2(-1)\end{bmatrix}=\begin{bmatrix}2+4 & 4-4\\1+2 & 2-2\end{bmatrix}=\begin{bmatrix}6 & 0\\3 & 0\end{bmatrix}$$

and

$$AC=\begin{bmatrix}2 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}5 & 6\\-1 & -3\end{bmatrix}$$

$$=\begin{bmatrix}2(5)+4(-1) & 2(6)+4(-3)\\1(5)+2(-1) & 1(6)+2(-3)\end{bmatrix}=\begin{bmatrix}10-4 & 12-12\\5-2 & 6-6\end{bmatrix}=\begin{bmatrix}6 & 0\\3 & 0\end{bmatrix}$$

Hence $AB = AC$ where $B \neq C$

-:10.17:-

If

$$\mathbf{A}=\begin{bmatrix}3 & 1 & 6\\2 & 1 & 0\\1 & 2 & 3\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}4 & 1 & 1\\2 & 1 & 3\\5 & 2 & 1\end{bmatrix}$$

Find AB and BA.

SOLUTION

$$AB=\begin{bmatrix}3 & 1 & 6\\2 & 1 & 0\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}4 & 1 & 1\\2 & 1 & 3\\5 & 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3(4)+1(2)+6(5) & 3(1)+1(1)+6(2) & 3(1)+1(3)+6(1)\\2(4)+1(2)+0(5) & 2(1)+1(1)+0(2) & 2(1)+1(3)+0(1)\\1(4)+2(2)+3(5) & 1(1)+2(1)+3(2) & 1(1)+2(3)+3(1)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+30 & 3+1+12 & 3+3+6\\8+2+0 & 2+1+0 & 2+3+0\\4+4+15 & 1+2+6 & 1+6+3\end{bmatrix}=\begin{bmatrix}44 & 16 & 12\\10 & 3 & 5\\23 & 9 & 10\end{bmatrix}$$

$$BA=\begin{bmatrix}4&1&1\\2&1&3\\5&2&1\end{bmatrix}\begin{bmatrix}3&1&6\\2&1&0\\1&2&3\end{bmatrix}$$

$$=\begin{bmatrix}4(3)+1(2)+1(1)&4(1)+1(1)+1(2)&4(6)+1(0)+1(3)\\2(3)+1(2)+3(1)&2(1)+1(1)+3(2)&2(6)+1(0)+3(3)\\5(3)+2(2)+1(1)&5(1)+2(1)+1(2)&5(6)+2(0)+1(3)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+1&4+1+2&24+0+3\\6+2+3&2+1+6&12+0+9\\15+4+1&5+2+2&30+0+3\end{bmatrix}=\begin{bmatrix}15&7&27\\11&9&21\\20&9&33\end{bmatrix}$$

-:10.18:-

If

$$A=\begin{bmatrix}3&1&3\\2&1&6\end{bmatrix}\quad B=\begin{bmatrix}4&1&6&8\\2&1&7&1\\3&2&2&2\end{bmatrix}$$

Find AB. Does the product BA exist?

SOLUTION

$$AB=\begin{bmatrix}3&1&3\\2&1&6\end{bmatrix}\begin{bmatrix}4&1&6&8\\2&1&7&1\\3&2&2&2\end{bmatrix}$$

$$=\begin{bmatrix}3(4)+1(2)+3(3)&3(1)+1(1)+3(2)&3(6)+1(7)+3(2)&3(8)+1(1)+3(2)\\2(4)+1(2)+6(3)&2(1)+1(1)+6(2)&2(6)+1(7)+6(2)&2(8)+1(1)+6(2)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+9&3+1+6&18+7+6&14+1+6\\8+2+18&2+1+12&12+7+12&16+1+12\end{bmatrix}=\begin{bmatrix}23&10&31&31\\28&15&31&29\end{bmatrix}$$

Multiplication of two matrices is only be possible if the number of columns of first matrix is equal to the number of rows in the second matrix. Here the number of columns of B and number of rows of A are not equal. The product BA does not exist.

-:10.19:-

If $A=\begin{bmatrix}3&1\\2&2\end{bmatrix}\quad B=\begin{bmatrix}2&1\\6&2\end{bmatrix}\quad C=\begin{bmatrix}3&3\\2&1\end{bmatrix}$

Show that A(B+C) = AB+AC

SOLUTION

$$A=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\quad B=\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}\quad C=\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}$$

$$B+C=\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}+\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}=\begin{bmatrix}2+3 & 1+3\\6+2 & 2+1\end{bmatrix}=\begin{bmatrix}5 & 4\\8 & 3\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}5 & 4\\8 & 3\end{bmatrix}=\begin{bmatrix}3(5)+1(8) & 3(4)+1(3)\\2(5)+2(8) & 3(4)+2(3)\end{bmatrix}$$

$$=\begin{bmatrix}15+8 & 12+3\\10+16 & 8+6\end{bmatrix}=\begin{bmatrix}23 & 15\\26 & 14\end{bmatrix}$$

$$AB=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}=\begin{bmatrix}3(2)+1(6) & 3(1)+1(2)\\2(2)+2(6) & 2(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}6+6 & 3+2\\4+12 & 2+4\end{bmatrix}=\begin{bmatrix}12 & 5\\16 & 6\end{bmatrix}$$

$$AC=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}=\begin{bmatrix}9+2 & 9+1\\6+4 & 6+2\end{bmatrix}$$

$$=\begin{bmatrix}11 & 10\\10 & 8\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}12 & 5\\16 & 6\end{bmatrix}+\begin{bmatrix}11 & 10\\10 & 8\end{bmatrix}=\begin{bmatrix}12+11 & 5+10\\16+10 & 6+8\end{bmatrix}=\begin{bmatrix}23 & 15\\26 & 14\end{bmatrix}$$

Hence A(B+C) = AB+AC

-:10.20:-

If

$$\mathbf{A}=\begin{bmatrix}\mathbf{3} & \mathbf{1}\\\mathbf{2} & \mathbf{0}\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}\mathbf{4} & \mathbf{-1}\\\mathbf{2} & \mathbf{3}\end{bmatrix}$$

Then find BA

SOLUTION

$$A=\begin{bmatrix}3 & 1\\2 & 0\end{bmatrix}\quad B=\begin{bmatrix}4 & -1\\2 & 3\end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4(3)+(-1)(2) & 4(1)+(-1)(0) \\ 2(3)+3(2) & 3(1)+3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 12-3 & 4+0 \\ 6+6 & 2+0 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 12 & 2 \end{bmatrix}$$

-:10.21:-

If

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & 2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

Then find a and b

SOLUTION

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix}$$

$$= \begin{bmatrix} 2(a)+3(7) & 2(2)+3(b) \\ 4(a)+5(7) & 4(2)+5(b) \end{bmatrix} = \begin{bmatrix} 2a+21 & 4+3b \\ 4a+35 & 8+5b \end{bmatrix}$$

We know that

$$= \begin{bmatrix} 2(a)+21 & 4+3b \\ 4a+35 & 8+5b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

We can put the corresponding values as

$2a+21=31$ or $4a+35=55$

$2a=31-21=10$ $4a=55-35=20$

$a=\frac{10}{2}=5$ $a=\frac{20}{4}=5$

Similarly

$4+3b=1$ or $8+5b=3$

$3b=1-4=-3$ $5b=3-8=-5$

$b=-1$ $b=1$

Hence a = 5, b = -1

-:10.22:-

If $A=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}$ $B=\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$

Then find (i) A+B (ii) A-B (iii) AB

SOLUTION

$$A=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix} \quad B=\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$$

(i)

$$A+B=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}+\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}=\begin{bmatrix}1+2&4+1&3+2\\2+0&1+4&8+8\\1+6&1+1&2+4\end{bmatrix}=\begin{bmatrix}3&5&5\\2&5&16\\7&2&6\end{bmatrix}$$

(ii)

$$A-B=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}-\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}+\begin{bmatrix}-2&-1&-2\\0&-4&-8\\-6&-1&-4\end{bmatrix}$$

$$=\begin{bmatrix}1-2&4-1&3-2\\2+0&1-4&8-8\\1-6&1-1&2-4\end{bmatrix}=\begin{bmatrix}-1&3&1\\2&-3&0\\-5&0&-2\end{bmatrix}$$

(iii)

$$AB=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$$

$$=\begin{bmatrix}1(2)+4(0)+3(6)&1(1)+4(4)+3(1)&1(2)+4(8)+3(4)\\2(2)+1(0)+8(6)&2(1)+1(4)+8(1)&2(2)+1(8)+9(4)\\1(2)+1(0)+2(6)&1(1)+1(4)+2(1)&1(2)+1(8)+2(4)\end{bmatrix}$$

$$=\begin{bmatrix}2+0+18&1+16+3&2+32+12\\4+0+48&2+4+8&4+8+32\\2+0+12&1+4+12&2+8+8\end{bmatrix}=\begin{bmatrix}20&20&46\\52&14&44\\14&17&18\end{bmatrix}$$

-:10.23:-

If

$$A=\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}\quad B=\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}$$

Then find (i) 2A+4B (ii) AB

SOLUTION

(i) $2A+4B$

$$2A=2\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}=\begin{bmatrix}10&8&6\\12&6&2\\16&18&4\end{bmatrix}$$

$$4B=4\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}=\begin{bmatrix}4&12&16\\8&16&20\\12&4&24\end{bmatrix}$$

$$2A+4B=\begin{bmatrix}10&8&6\\12&6&2\\16&18&4\end{bmatrix}+\begin{bmatrix}4&12&16\\8&16&20\\12&4&24\end{bmatrix}$$

$$=\begin{bmatrix}10+4&8+12&6+16\\12+8&6+16&2+20\\16+12&18+4&4+24\end{bmatrix}=\begin{bmatrix}14&20&22\\20&22&22\\28&22&28\end{bmatrix}$$

(ii) AB

$$AB=\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}$$

$$=\begin{bmatrix}5(1)+4(2)+3(3)&5(3)+4(4)+3(1)&5(4)+4(5)+3(6)\\6(1)+3(2)+1(3)&6(3)+3(4)+1(1)&6(4)+3(5)+1(6)\\8(1)+9(2)+2(3)&8(3)+9(4)+2(1)&8(4)+9(5)+2(6)\end{bmatrix}$$

$$=\begin{bmatrix}5+8+9&15+16+3&20+20+18\\6+6+3&18+12+1&24+15+6\\8+18+6&24+36+2&32+45+12\end{bmatrix}=\begin{bmatrix}22&34&58\\15&31&45\\32&62&89\end{bmatrix}$$

-:10.24:-

Find (a) AB (b) BA, given

$$A=\begin{bmatrix}7 & 7\\ 6 & 2\\ 1 & 8\end{bmatrix} \quad B=\begin{bmatrix}-3 & 9 & 1\\ 2 & 12 & 7\end{bmatrix}$$

SOLUTION

(a) AB

$$AB=\begin{bmatrix}7 & 7\\ 6 & 2\\ 1 & 8\end{bmatrix}\begin{bmatrix}-3 & 9 & 1\\ 2 & 12 & 7\end{bmatrix}$$

$$AB=\begin{bmatrix}7 & 7\\ 6 & 2\\ 1 & 8\end{bmatrix}\begin{bmatrix}-3 & 9 & 1\\ 2 & 12 & 7\end{bmatrix}=\begin{bmatrix}7(-3)+7(2) & 7(9)+7(12) & 7(1)+7(7)\\ 6(-3)+2(2) & 6(9)+2(12) & 6(1)+2(7)\\ 1(-3)+8(2) & 1(9)+8(12) & 1(1)+8(7)\end{bmatrix}$$

$$=\begin{bmatrix}-21+14 & 63+84 & 7+49\\ -18+4 & 54+24 & 6+14\\ -3+16 & 9+96 & 1+56\end{bmatrix}=\begin{bmatrix}-7 & 147 & 56\\ -14 & 78 & 20\\ 13 & 105 & 57\end{bmatrix}$$

(b) BA

$$BA=\begin{bmatrix}-3 & 9 & 1\\ 2 & 12 & 7\end{bmatrix}\begin{bmatrix}7 & 7\\ 6 & 2\\ 1 & 8\end{bmatrix}=\begin{bmatrix}-3(7)+9(6)+1(1) & -3(7)+9(2)+1(8)\\ 2(7)+12(6)+7(1) & 2(7)+12(2)+7(8)\end{bmatrix}$$

$$=\begin{bmatrix}-21+54+1 & -21+18+8\\ 14+72+7 & 14+24+56\end{bmatrix}=\begin{bmatrix}34 & 5\\ 93 & 94\end{bmatrix}$$

-:10.25:-

Find (a) AB (b) BA, given

$$A=\begin{bmatrix}4 & 9 & 8\\ 7 & 6 & 2\\ 1 & 5 & 3\end{bmatrix} \quad B=\begin{bmatrix}1 & 2 & 0\\ 5 & 3 & 1\\ 0 & 2 & 4\end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1)+9(5)+8(0) & 4(2)+9(3)+8(2) & 4(0)+9(1)+8(4) \\ 7(1)+6(5)+2(0) & 7(2)+6(3)+2(2) & 7(0)+5(1)+2(4) \\ 1(1)+5(5)+3(0) & 1(2)+5(3)+3(2) & 1(0)+5(1)+3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+45+0 & 8+27+16 & 0+9+32 \\ 7+30+0 & 14+18+4 & 0+6+8 \\ 1+25+0 & 2+15+6 & 0+5+12 \end{bmatrix} = \begin{bmatrix} 49 & 51 & 31 \\ 37 & 36 & 14 \\ 26 & 23 & 17 \end{bmatrix}$$

(b) BA

$$BA = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1(4)+2(7)+0(1) & 1(9)+2(6)+0(5) & 1(8)+2(2)+0(3) \\ 5(4)+3(7)+1(1) & 5(9)+3(6)+1(5) & 5(8)+3(2)+1(3) \\ 0(4)+2(7)+4(1) & 0(9)+2(6)+4(5) & 0(8)+2(2)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} 4+14+0 & 9+12+0 & 8+4+0 \\ 20+21+1 & 45+18+5 & 40+6+3 \\ 0+14+4 & 0+12+20 & 0+4+12 \end{bmatrix} = \begin{bmatrix} 18 & 21 & 12 \\ 42 & 68 & 49 \\ 18 & 32 & 16 \end{bmatrix}$$

-:10.26:-

Find (a) AB (b) BA, for a case where B is an identity matrix, given.

$$A = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 23(1)+6(0)+14(0) & 23(0)+6(1)+14(0) & 23(0)+6(0)+14(1) \\ 18(1)+12(0)+9(0) & 18(0)+12(0)+9(0) & 18(0)+12(0)+9(1) \\ 24(1)+2(0)+6(0) & 24(0)+2(1)+6(0) & 24(0)+2(0)+6(1) \end{bmatrix}$$

$$= \begin{bmatrix} 23+0+0 & 0+6+0 & 0+0+14 \\ 18+0+0 & 0+12+0 & 0+0+9 \\ 24+0+0 & 0+2+0 & 0+0+6 \end{bmatrix} = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}$$

(b) BA

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1(23)+0(18)+0(24) & 1(6)+0(12)+0(2) & 1(14)+0(9)+0(6) \\ 0(23)+1(18)+0(24) & 0(6)+1(12)+0(2) & 0(14)+1(9)+0(6) \\ 0(23)+0(18)+1(24) & 0(6)+0(12)+1(2) & 0(14)+0(9)+1(6) \end{bmatrix}$$

$$= \begin{bmatrix} 23+0+0 & 6+0+0 & 14+0+0 \\ 0+18+0 & 0+12+0 & 0+9+0 \\ 0+0+24 & 0+0+2 & 0+0+6 \end{bmatrix} = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}$$

-:10.27:-

Find the determinants of the following matrices.

i) $\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ **ii)** $\mathbf{B} = \begin{bmatrix} c & m \\ n & p \end{bmatrix}$

iii) $\mathbf{C} = \begin{bmatrix} 6 & 2 \\ 4 & 1 \end{bmatrix}$ **iv)** $\mathbf{D} = \begin{bmatrix} 15 & 8 \\ 3 & 4 \end{bmatrix}$

SOLUTION

(i)

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}, \text{ then } |A| = \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} = 12 - 20 = -8$$

(ii)

$$B = \begin{bmatrix} c & m \\ n & p \end{bmatrix}, \text{ then } |B| = \begin{vmatrix} c & b \\ n & p \end{vmatrix} = cp - mn$$

(iii)

$$C=\begin{bmatrix}6 & 2\\ 4 & 1\end{bmatrix}, \text{ then } |C|=\begin{vmatrix}6 & 2\\ 4 & 1\end{vmatrix}=6-8=-2$$

(iv)

$$D=\begin{bmatrix}15 & 8\\ 3 & 4\end{bmatrix}, \text{ then } |D|=\begin{vmatrix}15 & 8\\ 3 & 4\end{vmatrix}=60-24=36$$

-:10.28:-

Find the value of x when

i) $\begin{bmatrix}8 & x\\ 2 & 4\end{bmatrix}$ **is a singular matrix**

ii) $\begin{bmatrix}2 & 1\\ 3 & x\end{bmatrix}$ **is a singular**

SOLUTION

$$\begin{bmatrix}8 & x\\ 2 & 4\end{bmatrix} \text{ is a singular}$$

$$\text{then } \begin{vmatrix}8 & x\\ 2 & 4\end{vmatrix}=0$$

$$32-2x=0$$

$$-2x=-32$$

$$2x=32$$

$$x=16$$

-:10.29:-

Find the inverse of the following matrices.

i) $A=\begin{bmatrix}4 & 6\\ 10 & 8\end{bmatrix}$ **ii)** $B=\begin{bmatrix}-3 & -27\\ -6 & -18\end{bmatrix}$

iii) $C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **iv)** $D=\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$

SOLUTION

(i)

$$A=\begin{bmatrix}4 & 6\\ 10 & 8\end{bmatrix}, \quad |A|=\begin{vmatrix}4 & 6\\ 10 & 8\end{vmatrix}=32-60=-28$$

$$A^{-1} = \frac{1}{28}\begin{bmatrix} 4 & 6 \\ 10 & 8 \end{bmatrix} = \begin{bmatrix} \frac{-8}{28} & \frac{6}{28} \\ \frac{10}{28} & \frac{4}{28} \end{bmatrix} = \begin{bmatrix} \frac{-4}{14} & \frac{3}{14} \\ \frac{5}{14} & \frac{2}{14} \end{bmatrix}$$

(ii)

$$B = \begin{bmatrix} -3 & -27 \\ -6 & -18 \end{bmatrix}, \quad |B| = \begin{vmatrix} -3 & -27 \\ -6 & -18 \end{vmatrix} = 54 - 162 = -108$$

$$B^{-1} = -\frac{1}{108}\begin{bmatrix} -18 & 27 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} \frac{18}{108} & \frac{-27}{108} \\ \frac{-6}{108} & \frac{3}{108} \end{bmatrix} = \begin{bmatrix} \frac{6}{36} & \frac{-9}{36} \\ \frac{-2}{36} & \frac{1}{36} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{4} \\ \frac{-1}{18} & \frac{1}{36} \end{bmatrix}$$

(iii)

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |C| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$C^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(iv)

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad |D| = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3 - 0 = 3$$

$$D^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

-:10.30:-

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Prove that **(i)** $AA^{-1} = A^{-1}A = I$

(ii) $BB^{-1} = B^{-1}B$ **and**

(iii) $B^2 = I$

SOLUTION

(i)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}, \quad |A| = \begin{vmatrix} 2 & 1 \\ 0 & 8 \end{vmatrix} = 16 - 0 = 16$$

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 8 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix} = \begin{bmatrix} \frac{16}{16}+0 & \frac{-2}{16}+\frac{2}{16} \\ 0+0 & 0+\frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} \frac{16}{16}+0 & \frac{8}{16}-\frac{8}{16} \\ 0+0 & 0+\frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence $AA^{-1} = A^{-1}A$

(ii)

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |B| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$B^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(iii)

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

-:10.31:-

Solve the following sets of equations with the help of matrices.

a) $x + y = 8$, $2x - y = 7$

b) $2x + 5y = 19$, $x + 3y = 11$

c) $3x + 2y = 1$, $5x - 3y = 27$

d) $3x + 2y = 12$, $x + 5y = 17$

e) $2x - 3y = 1$
$x + 4y = 6$

f) $7x - 3y = 3$
$2x + y = 2$

SOLUTION

(a)
$$x + y = 8$$
$$2x - y = 7$$

Here

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

So

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3 \neq 0$$

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} + \frac{7}{3} \\ \frac{16}{3} - \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{15}{3} \\ \frac{9}{3} \end{bmatrix} = \begin{matrix} 5 \\ 3 \end{matrix}$$

Therefore $x = 5, y = 3$

(b)
$$2x + 5y = 19$$
$$x + 3y = 11$$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1 \neq 0$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ 11 \end{bmatrix} = \begin{bmatrix} 57-55 \\ -19+22 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore. $x = 2$, $y = 3$

(c) $3x + 2y = 1$

$5x - 3y = 27$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} = -9-10 = -19 \neq 0$$

$$A^{-1} = \frac{-1}{19}\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}\begin{bmatrix} 1 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} + \frac{54}{19} \\ \frac{5}{19} - \frac{-81}{19} \end{bmatrix} = \begin{bmatrix} \frac{57}{19} \\ -\frac{76}{19} \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Therefore, $x = 3$, $y = -4$

(d) $3x + 2y = 12$

$x + 5y = 17$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 17 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = 15-2 = 13 \neq 0$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{bmatrix}\begin{bmatrix} 12 \\ 17 \end{bmatrix} = \begin{bmatrix} \frac{60}{13} - \frac{34}{13} \\ -\frac{12}{13} + \frac{51}{13} \end{bmatrix} = \begin{bmatrix} \frac{26}{13} \\ \frac{39}{13} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 2, y = 3$

(e) $2x - 3y = 1$

$x + 4y = 6$

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} = 8 + 3 = 11$$

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} + \frac{18}{11} \\ \frac{-1}{11} + \frac{12}{11} \end{bmatrix} = \begin{bmatrix} \frac{22}{11} \\ \frac{11}{11} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Therefore, $x = 2, y = 1$

(f) $7x - 3y = 3$

$2x + y = 2$

Here

$$A = \begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$
$$\begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & -3 \\ 2 & 1 \end{vmatrix} = 7 + 6 = 13$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 1 & 3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} + \frac{6}{13} \\ \frac{-6}{13} + \frac{14}{13} \end{bmatrix} = \begin{bmatrix} \frac{9}{13} \\ \frac{8}{13} \end{bmatrix}$$

Therefore, $x = {}^{9}/_{13}$, $y = {}^{8}/_{13}$

-:10.32:-

Solve the following sets of equations with the help of matrices.

a) (i) $2x - 6y = -12$, $3x - 2y = -4$ **(ii)** $\frac{3}{2}x + y = \frac{3}{4}$, $x - 2y = 2$

SOLUTION

(a)

(i)
$$2x - 6y = 12$$
$$3x - 2y = -4$$

Here

$$A = \begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & -6 \\ 3 & -2 \end{vmatrix} = -4 + 18 = 14 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{14}\begin{bmatrix} -2 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{bmatrix}\begin{bmatrix} -12 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{24}{14} - \frac{24}{14} \\ \frac{36}{14} - \frac{8}{14} \end{bmatrix} = \begin{bmatrix} \frac{0}{14} \\ \frac{28}{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Therefore $x = 0,\ y = 2$

(ii)
$$\frac{3}{2}x + y = \frac{3}{4}$$
$$x - 2y = 2$$

Here $A = \begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$

So
$$\begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{vmatrix} = -3 - 1 = -4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = -\frac{1}{4}\begin{bmatrix} -2 & -1 \\ -1 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{8} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{8} \end{bmatrix}\begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} + \frac{1}{2} \\ \frac{3}{16} - \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{3+4}{8} \\ \frac{3-12}{16} \end{bmatrix} = \begin{bmatrix} \frac{7}{8} \\ \frac{-9}{16} \end{bmatrix}$$

Therefore $x = {}^{7}/_{8}, y = {}^{-9}/_{16}$

(b)

$$2x + 3y = 10$$
$$4x + 8y = 24$$

Here

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{4} & \frac{-3}{4} \\ \frac{-4}{4} & \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 10 \\ 24 \end{bmatrix} = \begin{bmatrix} 20 - 18 \\ -10 + 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Therefore $x = 2, y = 2$

(c)

$$3x + 2y = 12$$
$$4x + 5y = 23$$

Here

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

So

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 15 - 8 = 7 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & \frac{-2}{7} \\ \frac{-4}{7} & \frac{3}{7} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} \frac{5}{7} & \frac{-2}{7} \\ \frac{-4}{7} & \frac{3}{7} \end{bmatrix}\begin{bmatrix} 12 \\ 23 \end{bmatrix} = \begin{bmatrix} \frac{60}{7} - \frac{46}{7} \\ \frac{-48}{7} + \frac{69}{7} \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{21}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore $x = 2$, $y = 3$

-:10.33:-

Solve the following sets of equations by Cramer's Rule

a) $2x + 4y = 5$, $6x - y = 1$

b) $x + 3y = 3$, $2x + 5y = 7$

c) $x - 2y = 1$, $2x - 5y = 1$

d) $5x + 2y = 4$, $3x - 2y = 12$

SOLUTION

(a)

$$2x + 4y = 5$$
$$6x - y = 1$$

$$\begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}} = \frac{-5-4}{-2-24} = \frac{-9}{-26} = \frac{9}{26}$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 2 & 5 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}}=\frac{2-30}{-2-24}=\frac{-28}{-26}=\frac{14}{13}$$

Hence x = 9/26 and y = 14/13

(b)

$$x+3y=3$$
$$2x+5y=7$$
$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 3 & 3 \\ 7 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}}=\frac{15-21}{5-6}=\frac{-6}{-1}=6$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}}=\frac{7-6}{5-6}=\frac{-1}{-1}=-1$$

Hence x = 6 and y = –1

(c)

$$x-2y=1$$
$$2x-5y=1$$
$$\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 1 & -2 \\ 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix}}=\frac{-5+2}{-5+4}=\frac{3}{-1};$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix}}=\frac{1-2}{-5+4}=\frac{-1}{-1}=1$$

Hence $x = 3$ and $y = 1$

(d)

$$5x + 2y = 4$$
$$3x - 2y = 1$$

$$\begin{bmatrix} 5 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 5 & 2 \\ 3 & -2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\begin{bmatrix} 4 \\ 12 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 4 & 2 \\ 12 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2 \end{vmatrix}}=\frac{-8-24}{-10-6}=\frac{-32}{-16}=2$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 5 & 4 \\ 3 & 12 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2 \end{vmatrix}}=\frac{60-12}{-10-6}=\frac{48}{-16}=-3$$

Hence $x = 2$ and $y = -3$